

THE DERIVE - NEWSLETTER #128

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THE BULLETIN OF THE



USER GROUP

+ CAS-TI

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You can download the full collection of David Halprin's papers from academia.edu

<https://independent.academia.edu/DavidHalprin>

Other websites which might be interesting:

Clark Kimberling's Encyclopedia of Triangle Centers – ETC

<https://faculty.evansville.edu/ck6/encyclopedia/ETC.html>

Paper from Simon Plouffe:

<http://plouffe.fr/Inverseofprimes/La%20forme%20de%20bn%20mod%20p.pdf>

Explore great Graphics:

<https://lengler.dev/TimesTableWebGL/>

Families of Functions Modular Course (TI)

<https://education.ti.com/en/resources/family-of-functions>

Notice and Wonder with TI Tools

<https://sites.google.com/view/fofwonder/home>

Understanding Probability (Five YouTube presentations)

<https://www.youtube.com/playlist?list=PLRdShFxAjBLhuBWDB3lyCv9n6jiNFk4J>

Michael de Villiers' Dynamic Mathematics Learning website

<http://dynamicmathematicslearning.com/homepage4.html>

A collection of tasks for German end examination mathematics and other subjects

<https://www.iqb.hu-berlin.de/abitur/sammlung/mathematik/>

New STEM books K-12

<https://www.nsta.org/bsb23>

See also page 56.

Question from Ross Barnes: Did anyone every compile a bug list for Derive 6.10?

Dear DUG Members,

It really happened as I announced in the last DNL. The last newsletter from 2022 was not finished until 2023. I had planned to include a few more contributions, but it became "only" three, but these are very detailed.

In the tasks of Michel Beaudin, the treatment of the not so well-known *LambertW*-function should find your interest. It kept me very busy at least and led to an intensive communication with Michel. In the MTH128.zip file you can find an update of Michel's extensive library for TI-Nspire including description, thanks Michel.

Some time ago (TI-92 time), Tania Koller was part of an international project group. In this context, she conducted a survey of her students about their use of video games with the intention of processing the data obtained in various ways with the students. She had a lot of success with her students and I would like to thank Tania for the permission to translate her extensive material and include it in the DNL.

Maybe this can also be an incentive for you to ask your students about their habits with smartphones, social media, leisure activities, ... and thus gain realistic material for statistics lessons?

I came across the "Way of the Spider" via the beautiful puzzles regularly published by Heinrich Hemme at www.spektrum.de/. I have extended this puzzle, the solution of which is very simple, with a three-dimensional question – which is not so elementary.

As so often before, an exchange with Wolfgang Alvermann resulted, who then also worked on this problem. His German version is included in MTH128.zip. The small picture shows a paper model of the 3D-task in front of the DERIVE solution on my PC.

The User Forum also became very extensive this time and for once only concerns DERIVE topics.

Please note the numerous links and the Ross Barnes' question on the first inside page.

I remain with my best wishes as ever
Yours

Liebe DUG-Mitglieder,

es ist wirklich so gekommen, wie ich im letzten DNL angekündigt habe. Der letzte Newsletter aus 2022 wurde erst 2023 fertig. Ich hatte schon vor, einige Beiträge mehr unterzubringen, es sind aber „nur“ drei geworden, diese aber sehr ausführlich.

In den Aufgaben von Michel Beaudin soll die Behandlung der nicht so bekannten *LambertW*-Funktion Ihr Interesse finden. Mich hat sie sehr beschäftigt und zu einer intensiven Kommunikation mit Michel geführt. In der MTH128.zip-Datei könnt Ihr ein Update von Michels umfangreicher Bibliothek für TI-Nspire samt Beschreibung finden, danke Michel.

Tania Koller war vor einiger Zeit (TI-92 Zeit!) Teil einer internationalen Projektgruppe. In diesem Rahmen hat sie eine Befragung ihrer Schüler über deren Umgang mit Videospielen durchgeführt mit der Absicht, die so gewonnenen Daten auf verschiedene Weise mit den Schülern aufzuarbeiten. Sie hatte damit bei ihren Schülern viel Erfolg. Ich bedanke mich bei Tania für die Erlaubnis, ihr umfangreiches Material übersetzen und in den DNL aufnehmen zu dürfen.

Vielleicht kann dies auch ein Anreiz für Euch sein, Eure Schüler über die Gewohnheiten mit Smartphone, Social Media, Freizeitaktivitäten, ... zu befragen und so realistisches Material für den Statistikerunterricht zu gewinnen?

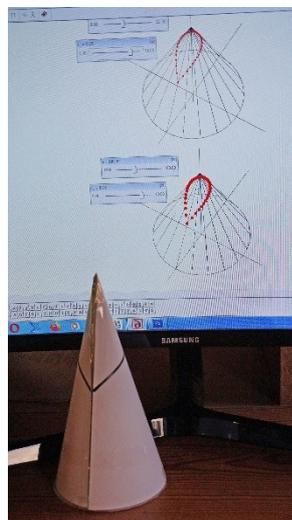
Auf den „Weg der Spinne“ bin ich über die schönen Rätsel gestoßen, die regelmäßig von Heinrich Hemme auf www.spektrum.de/ veröffentlicht werden. Ich habe dieses Rätsel, dessen Lösung sehr einfach ist, mit einer dreidimensionalen Fragestellung erweitert – die nicht so elementar ist.

Wie schon so oft ergab sich ein Austausch mit Wolfgang Alvermann, der dann dieses Rätsel auch bearbeitete. Seine deutschsprachige Version ist in MTH128.zip enthalten. Das kleine Bild zeigt ein Papiermodell der 3D-Aufgabe vor der DERIVE-Lösung auf meinem PC.

Bitte beachtet die zahlreichen Links und die Frage von Ross Barnes auf der ersten Innenseite.

Ich verbleibe wie immer mit den besten Wünschen

Euer



The *DERIVE-NEWSLETTER* is the Bulletin of the *DERIVE & CAS-TI User Group*. It is published at least four times a year with a content of 40 pages minimum. The goals of the *DNL* are to enable the exchange of experiences made with *DERIVE*, *TI-CAS* and other CAS as well to create a group to discuss the possibilities of new methodical and didactical manners in teaching mathematics.

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Contributions:

Please send all contributions to the Editor. Non-English speakers are encouraged to write their contributions in English to reinforce the international touch of the *DNL*. It must be said, though, that non-English articles will be warmly welcomed nonetheless. Your contributions will be edited but not assessed. By submitting articles, the author gives his consent for reprinting it in the *DNL*. The more contributions you will send, the more lively and richer in contents the *DERIVE & CAS-TI Newsletter* will be.

Next issue:

March 2023

Preview: Contributions waiting to be published

Some simulations of Random Experiments, J. Böhm, AUT, Lorenz Kopp, GER
 Wonderful World of Pedal Curves, J. Böhm, AUT
 Simulating a Graphing Calculator in *DERIVE*, J. Böhm, AUT
 Cubics, Quartics – Interesting features, T. Koller & J. Böhm, AUT
 Logos of Companies as an Inspiration for Math Teaching
 Exciting Surfaces in the FAZ, BooleanPlots.mth, P. Schofield, UK
 Old traditional examples for a CAS – What's new? J. Böhm, AUT
 Mandelbrot and Newton with *DERIVE*, Roman Hašek, CZ
 Tutorials for the NSpireCAS, G. Herweyers, BEL
 Dirac Algebra, Clifford Algebra, Vector-Matrix-Extension, D. R. Lunsford, USA
 Another Approach to Taylor Series, D. Oertel, GER
 Charge in a Magnetic Field, H. Ludwig, GER
 More Applications of TI-Innovator™ Hub and TI-Innovator™ Rover
 Surfaces and their Duals, Cayley Symmetroid, J. Böhm, AUT
 A Collection of Special Problems, W. Alvermann, GER
 DERIVE Bugs? D. Welz, GER
 Tweening & Morphing with TI-NspireCX-II-T, J. Böhm, AUT
 The Gap between Poor and Rich, J. Böhm, AUT
 More functions from M. Myers and from Bhuvanesh's Mathtools-library
 TaxiCab Conics, Two alternate Approaches to Conics, R. Haas, USA
 QR-Code light, Random numbers following a given distribution
 Properties of Polygons, Quartiles, F. de Jesús Martínez Vargas, Mexico
 Penalty Shootout Mathematics, B. Grabinger, GER
 153 is another Special Number, and others

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Here we continue Michel Beaudin's list of exercises connected with his Résumé 1 presented in earlier DNLs.

Exercise list for Summary 1 – Part 2

Problems 3 Equation solving and the *LambertW* function

Solve the following equations by hand so as to rewrite them in the form $ye^y = z$ whose solutions are known to be $y = W(k, z)$ where "*W*" is the *LambertW* function (k one of its branches). Note that the "solve" command in your symbolic system should find, at least numerically, the real solutions of the equations of questions b) to i). In connection with the LambertW function programmed in the Nspire kit_ets_mb.tns library^[*] or the one included in *Maple* or the ProductLog function in *Mathematica*, this allows you to check your answers for these exercises.

- a) Solve an equation of the type $ae^{bt} + ct + d = 0$ where $a, b, c,$ and d are four non-zero constants (possibly complex) and t is the variable with respect to which we solve. Note that this type of equation appears when we look for the time for an object in free fall to reach the ground and when we assume the air resistance force is proportional to the speed of the object.
- b) Find all real solutions of the equation $\ln(x) = 4 - x$.
- c) Find all real solutions of the equation $x^4 = 3^x$.
- d) Find all real solutions of the equation $e^{3x} = x^2 + 6x + 9$.
- e) Find all real (and positive) solutions of the equation $x^x = \frac{4}{5}$.
- f) Find all real and some complex solutions of the equation $\left(\frac{1}{2}\right)^x = x^8$.
- g) Solve the equation $x^6 = 1.1^{-x}$ and give all real and complex solutions. Check that the complex solution you found by Newton's method to problem 1 f) is included in one of the functions $W_0(z)$ and $W_{-1}(z)$ which are denoted by *LambertW*(0, z) and *LambertW*(-1, z) respectively in the kit_ets_mb library.
- h) Find all real solutions of the equation $2^x = x + 5$.
- i) Find all solutions of the equation $e^{-x^2} = x$. More generally, show that an equation of the form $ab^{x^c} = x^d$ can be solved via *LambertW*.

[*] For your convenience kit_ets_mb.tns is included in MTH128.zip.

Problem 3c)

Find all real solutions of the equation $x^4 = 3^x$.

A graph in a "standard" window shows two solutions but it is clear that a third solution exists since the exponential function 3^x will eventually dominate the power function x^4 because of the de l'Hospital rule. A value table also confirms this. See figure 8. Anyway, the Nspire solver finds the three approximate values very quickly. Let's find those three values.

The four unit roots are $1, 1, i$ and $-i$ but we are only looking for real solutions, so we can rewrite the equation as

$$(3^x)^{1/4} = \pm x.$$

Thus $3^{x/4} = \pm x$, hence $e^{\frac{x}{4}\ln(3)} = \pm x$, hence $xe^{-\frac{x}{4}\ln(3)} = \pm 1$, hence $-\frac{x}{4}\ln(3)e^{-\frac{x}{4}\ln(3)} = \pm \frac{\ln(3)}{4}$. We have thus

obtained an equation of the form $ye^y = z$ of which we know that the solutions are given by $y = \text{LambertW}(k, z) = "W(k, z)"$. Only the branches $k = 0$ and $k = 1$ provide real values when $z \geq -1/e \approx -0.367879$. And $-\ln(3)/4$ is about -0.274653 . So the three real solutions are

$$-\frac{4}{\ln(3)}W\left(0, \frac{\ln(3)}{4}\right), -\frac{4}{\ln(3)}W\left(0, -\frac{\ln(3)}{4}\right) \text{ and } -\frac{4}{\ln(3)}W\left(-1, -\frac{\ln(3)}{4}\right).$$

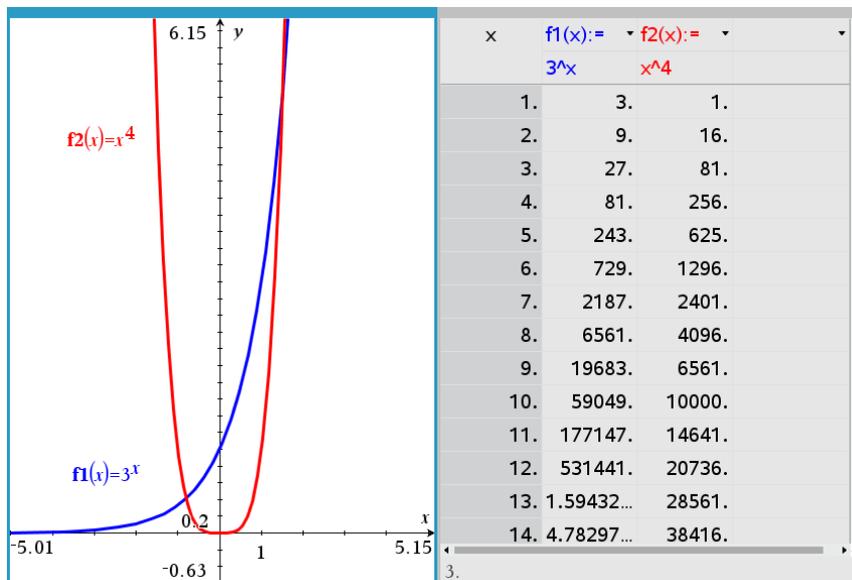


Figure 8

You can confirm these three values if the *LambertW* function is implemented in your symbolic system or has been programmed, which the author of these lines has done in Nspire:

$w(k, z) := \text{kit_ets_mb} \backslash \text{lambertw}(k, z)$	Done
$\frac{-4}{\ln(3)} \cdot w\left(0, \frac{\ln(3)}{4}\right)$	-0.802246
$\frac{-4}{\ln(3)} \cdot w\left(0, -\frac{\ln(3)}{4}\right)$	1.51678
$\frac{-4}{\ln(3)} \cdot w\left(-1, -\frac{\ln(3)}{4}\right)$	7.17476
$\Delta \text{ solve}(x^4 = 3^x, x)$	$x = -0.802246$ or $x = 1.51678$ or $x = 7.17476$

Figure 9

Remark: if we had asked to find *all* the solutions of the equation $x^4 = 3^x$, it would have been enough by using α to designate one of the 4 fourth roots of 1 to write that the equation is equivalent to

$$x e^{-\frac{x}{4} \ln(3)} = \alpha \Leftrightarrow -\frac{x}{4} \ln(3) e^{-\frac{x}{4} \ln(3)} = -\alpha \frac{\ln(3)}{4} \Leftrightarrow x = -\frac{4}{\ln(3)} W\left(k, -\alpha \frac{\ln(3)}{4}\right)$$

with α each taking one of the four values 1, i , $-i$ and -1 . Here are some complex solutions (Figure 10) and the graphs in Figure 11 of the real (red) and imaginary (blue) parts of the equation $(x + iy)^4 = 3^{x+iy}$.

$\frac{-4}{\ln(3)} \cdot w\left(0, \frac{i \cdot \ln(3)}{4}\right)$	$-0.231879 - 0.909196 \cdot i$
$\frac{-4}{\ln(3)} \cdot w\left(0, \frac{-i \cdot \ln(3)}{4}\right)$	$-0.231879 + 0.909196 \cdot i$
$\frac{-4}{\ln(3)} \cdot w\left(-1, \frac{i \cdot \ln(3)}{4}\right)$	$9.18461 + 8.42098 \cdot i$
$\frac{-4}{\ln(3)} \cdot w\left(-1, \frac{-i \cdot \ln(3)}{4}\right)$	$11.574 + 21.0462 \cdot i$
$\frac{-4}{\ln(3)} \cdot w\left(2, \frac{i \cdot \ln(3)}{4}\right)$	$14.002 - 44.6471 \cdot i$
$\frac{-4}{\ln(3)} \cdot w\left(2, \frac{-i \cdot \ln(3)}{4}\right)$	$12.9878 + 32.9481 \cdot i$

Figure 10

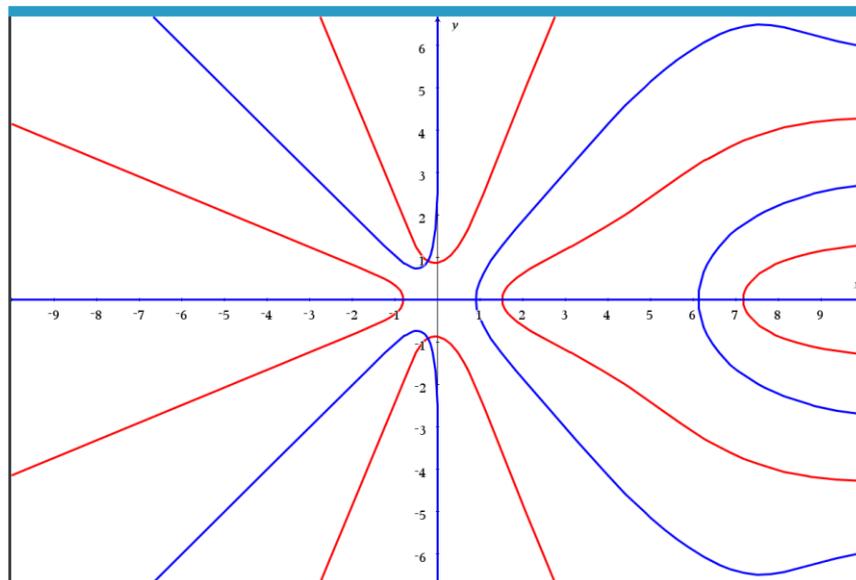


Figure 11

(See Michel's trick to produce the implicit plot of real and imaginary parts with TI-Nspire at the end of this contribution!)

The DERIVE result using the Lambertw-function, programmed following Michel's Nspire procedure:

$$-\frac{4}{\text{LOG}(3)} \cdot \left[\text{lambertw}\left(0, \frac{\text{LOG}(3)}{4}\right), \text{lambertw}\left(0, -\frac{\text{LOG}(3)}{4}\right), \text{lambertw}\left(-1, -\frac{\text{LOG}(3)}{4}\right) \right]$$

$[-0.8022464315, 1.516776412, 7.174755827]$

Problem 3d)

Solving equations and the *LambertW* function. Find all real solutions of the equation $e^{3x} = x^2 + 6x + 9$.

Figure 1 shows a screen capture of TI-Nspire which indicates that we will have three solutions. The lower right part sets the stage for using *LambertW*.

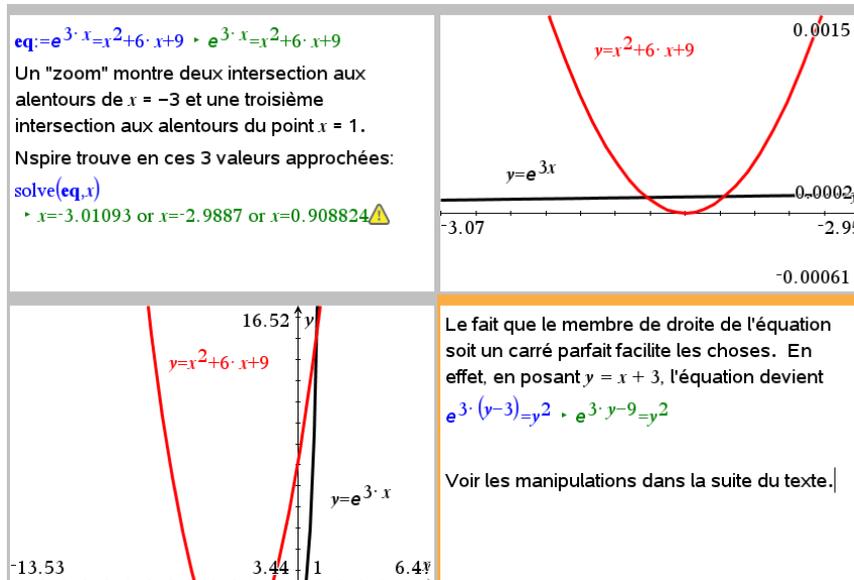


Figure 1

Change of variable $y = x + 3$ thus transforms the equation into $e^{3y}e^{-9} = y^2$. Taking the square root of each side, we have $e^{3y/2}e^{-9/2} = \pm y \Leftrightarrow -\frac{3}{2}ye^{-3y/2} = \pm \frac{3}{2}e^{-9/2}$. Notice that $-\frac{1}{e} < -\frac{3}{2}e^{-9/2} < 0$. Therefore we have $y = -\frac{2}{3}LambertW\left(0, \frac{3}{2}e^{-9/2}\right), -\frac{2}{3}LambertW\left(0, -\frac{3}{2}e^{-9/2}\right), -\frac{2}{3}LambertW\left(-1, -\frac{3}{2}e^{-9/2}\right)$.

Returning to "x", we then have the following three answers:

$$\begin{aligned} x &= -\frac{2}{3} \cdot \text{kit_ets_mb} \backslash \text{lambertw} \left(0, \frac{3}{2} \cdot e^{-\frac{9}{2}} \right) - 3 \rightarrow x = -3.01093 \\ x &= -\frac{2}{3} \cdot \text{kit_ets_mb} \backslash \text{lambertw} \left(0, -\frac{3}{2} \cdot e^{-\frac{9}{2}} \right) - 3 \rightarrow x = -2.9887 \\ x &= -\frac{2}{3} \cdot \text{kit_ets_mb} \backslash \text{lambertw} \left(-1, -\frac{3}{2} \cdot e^{-\frac{9}{2}} \right) - 3 \rightarrow x = 0.908824 \end{aligned}$$

Figure 2

Remark:

As Michel wrote above, he has produced a function $\text{lambertw}(k,w)$ for TI-Nspire. Since this is not a function to meet at every corner in mathematics, I felt challenged to program $\text{lambertw}(k,w)$ for DERIVE, too. This was not so easy and was the starting point for an interesting exchange of mails, functions and figures with Michel Beaudin.

From Josef to Michel

Dear Michel,

I am now working on DNL128 which will include some of your exercises.

In Chapter 3 you have examples involving the LambertW-function and you are using your great N-Spire library.

Do you think that it might make sense to transfer your function to DERIVE? MATHEMATICA's Product-Log function is always $W(0,x)$? What if I'd like to find $W(-1, \dots)$ or $W(2, \dots)$.

Sorry for my stupid question, but I am really not an expert for this function.

From Michel to Josef

Dear Josef, let me first explain why I have programmed LambertW in Nspire. When I was teaching MAT805, students who were coming from North Africa were not familiar with the use of CAS. The other ones (those from ETS or elsewhere in Quebec or those from France) knew (at least) Matlab (also Nspire in the case of those from ETS). So I told the North Africa students to download a trial version of Nspire because it is easy to use (compare to the "big CAS" Maple, Mathematica) and a contact at TI Canada was able to give me some activation codes for extending the trial version for one year (so long enough for the students to use Nspire during all the semester).

During an exam (I join my midterm exam of Winter 2022 for your information with the solutions done using Nspire), my students were asked to solve some equations whose answer involves LambertW. Because the "solve" command of Nspire does not contain LambertW, I have decided to program it because it was a way to check (=validate) their answers. And also it is still important for me to continue to use pencil and paper AND use CAS to validate answers.

But I don't think my programming is so good --even though I did not find bugs so far. If you want to "export" it to Derive, no problem with me. What can help you --and this is what I first looked at-- is the article about LambertW (mainly starting at page 342). It helped me a lot to program LambertW in Nspire.

https://www.researchgate.net/publication/2458364_On_the_Lambert_W_Function

From Josef to Michel

I started programming the function with DERIVE and came across an interesting fact.

You can find one call of LambertW(0,5i) on the Nspire and the same call with DERIVE.

They are different, but when checking the solutions for the system of equations both solutions seem to be correct?

<code>kit_ets_mb\lambertw(0,5·i)</code>	$1.19805+0.917331·i$
$\Delta \text{zeros} \left\{ \begin{array}{l} (x \cdot \cos(y) - y \cdot \sin(y)) \cdot e^x - \text{real}(5 \cdot i) \\ (x \cdot \sin(y) + y \cdot \cos(y)) \cdot e^x - \text{imag}(5 \cdot i) \end{array} \right\}, \{x,y=0\}$	$[1.19805 \quad 0.917331]$
$(x \cdot \sin(y) + y \cdot \cos(y)) \cdot e^x - \text{imag}(5 \cdot i) x=1.19805 \text{ and } y=0.917331$	0.00001
$(x \cdot \sin(y) + y \cdot \cos(y)) \cdot e^x - \text{imag}(5 \cdot i) x=-0.223373 \text{ and } y=6.247446$	0.000003

$\text{lambertw}(0, 5 \cdot i) = [-0.2233736303, 6.247446140]$

$$\left[(x \cdot \cos(y) - y \cdot \sin(y)) \cdot e^x - \text{RE}(w), (x \cdot \sin(y) + y \cdot \cos(y)) \cdot e^x - \text{IM}(w) \right]$$

$$\left[((-0.2233736303) \cdot \cos(6.24744614) - 6.24744614 \cdot \sin(6.24744614)) \cdot e^{-0.2233736303} - \text{RE}(5 \cdot i), ((-0.2233736303) \cdot \sin(6.24744614) + 6.24744614 \cdot \cos(6.24744614)) \cdot e^{-0.2233736303} - \text{IM}(5 \cdot i) \right]$$

$$\left[3.161375020 \cdot 10^{-9}, 0 \right]$$

$$\left[(1.19805 \cdot \cos(0.917331) - 0.917331 \cdot \sin(0.917331)) \cdot e^{1.19805} - \text{RE}(5 \cdot i), (1.19805 \cdot \sin(0.917331) + 0.917331 \cdot \cos(0.917331)) \cdot e^{1.19805} - \text{IM}(5 \cdot i) \right]$$

$[0.0000003125470258, 0.000009732000179]$

From Michel to Josef

Now, what you did in Derive yields $\text{LambertW}(1,5i)$ and not $\text{LambertW}(0,5i)$: I have also checked using Maple. Also, in a calculator page of Nspire, if you substitute the last answer into an expression, use "ans" instead of copying the answer (because you are "scratching some hidden decimals"). See the Nspire file "LambertW_check". Take care,

From Josef to Michel

Thanks for the attachments. I will exchange the old version of your library by the new one (and add a note in the next DNL!).

It is again an interesting communication – at least for me!

What concerns the LambertW function. I had no problem with the "accuracy" but I am still surprised that Nspire and DERIVE obtain different solutions for the system of equations.

I attach my DERIVE file. Entering $\text{lambertw}(0,5i)$ should lead to the last if-clause in my function (and it does, I checked). But the solutions are the same as obtained in the if-clause above!

From Michel to Josef

Dear Josef, when I used my "newtons2" function (which is a copy of Derive's Newtons but only for 2 variables), I get the same answer as Derive. So, I suspect that the "zeros" function of Nspire is not exactly the same function and this is why (I think) this yields different values. See the file.

kit_ets_mb\lambertw(0,5·i) → 1.19805+0.917331·i

$$u = (x \cdot \cos(y) - y \cdot \sin(y)) \cdot e^x \rightarrow (x \cdot \cos(y) - y \cdot \sin(y)) \cdot e^x$$

$$v = (x \cdot \sin(y) + y \cdot \cos(y)) \cdot e^x \rightarrow (x \cdot \sin(y) + y \cdot \cos(y)) \cdot e^x$$

zeros($\left\{ \begin{array}{l} u \\ v-5 \end{array} \right\}, \{x=0, y=0\}$) → [1.19805 0.917331] ⚠

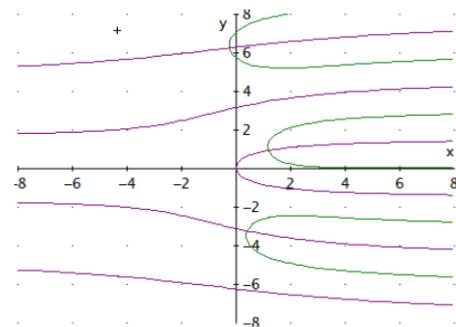
0.	0
0.	5.
-0.873195	5.78429
-0.074404	6.64119
-0.269828	6.31856
-0.225476	6.24433
-0.223375	6.24745
-0.223374	6.24745
-0.223374	6.24745
-0.223374	6.24745
-0.223374	6.24745
-0.223374	6.24745

kit_ets_mb\newtons2([u v-5],[x y],[0. 0.],10) →

There is still no real implicit 2D plotting in Nspire. Using good old Derive, on next page we can see 3 solutions (among the infinity) of the equation

$$z \cdot e^z = 0 + 5 \cdot i$$

We see in the second quadrant what Newtons yields. |



From Josef to Michel

here I am again. It was yesterday afternoon while I was watching any stuff on TV – in fact I was not really watching but had my thoughts in the next DNL. Suddenly I had a lightning in my brain – until then I did not realize the meaning of the parameter “k” in your LambertW function, since ProuctLog does without k.

I know that LambertW is multivalued for complex numbers but I didn’t read your sentence

“Only the branches $k = 0$ and $k = 1$ provide real values when $z \geq 1/e \approx -0.367879$. And $-\ln(3)/4$ is about -0.274653 .” with understanding. So, k is nothing else than a sort of key (switch) to get any of the many complex solutions. Is that right?

And for only a few cases ($\text{IM}(w) = 0$ and $\text{RE}(w) \geq -1/e \vee \text{RE}(w) < 0$ and $k = 0 \wedge \text{IM}(w) = 0 \wedge \text{RE}(w) \geq -1/e$) a real solution is possible which can be found using the key 0 or -1.

I must admit I found my idea confirmed by studying once more your program. I think, that I’ve got it now. Needed some time, Shame on me.

I add a word document with some results of my DERIVE LambertW program together with a possible LambertW2 program which can be used if the first one fails (because of the problems with the Newton Method procedure).

Many thanks for your patience and providing a chance for my brain to experience an “enlightenment”.

<code>kit_ets_mb\lambertw(0,0)</code>	0
<code>kit_ets_mb\lambertw(0,e)</code>	1
<code>kit_ets_mb\lambertw(0, $\frac{-\pi}{2}$)</code>	$\frac{\pi}{2} \cdot i$
<code>kit_ets_mb\lambertw(-1, $\frac{-\pi}{2}$)</code>	$\frac{-\pi}{2} \cdot i$
<code>kit_ets_mb\lambertw(1, $\pi \cdot i$)</code>	$\pi \cdot i$
<code>kit_ets_mb\lambertw(-1, $\pi \cdot i$)</code>	$-\pi \cdot i$
<code>kit_ets_mb\lambertw(1, $\frac{3 \cdot \pi}{2}$)</code>	$\frac{3 \cdot \pi}{2} \cdot i$
<code>kit_ets_mb\lambertw(-1, $\frac{-1}{e}$)</code>	-1
<code>kit_ets_mb\lambertw(0, $\frac{-1}{e}$)</code>	-1
<code>kit_ets_mb\lambertw(-1, $\frac{3 \cdot \pi}{2}$)</code>	$\frac{-3 \cdot \pi}{2} \cdot i$
<code>kit_ets_mb\lambertw(1,0)</code>	$-\infty$

<code>lambertw(0, 0) = 0</code>
<code>lambertw(0, e) = 1</code>
<code>lambertw(0, $-\frac{\pi}{2}$) = $\frac{\pi \cdot i}{2}$</code>
<code>lambertw(-1, $-\frac{\pi}{2}$) = $-\frac{\pi \cdot i}{2}$</code>
<code>lambertw(1, $-\pi \cdot i$) = $\pi \cdot i$</code>
<code>lambertw(-1, $\pi \cdot i$) = $-\pi \cdot i$</code>
<code>lambertw(1, $\frac{3 \cdot \pi}{2}$) = $\frac{3 \cdot \pi \cdot i}{2}$</code>
<code>lambertw(-1, $-\frac{1}{e}$) = -1</code>
<code>lambertw(0, $-\frac{1}{e}$) = -1</code>
<code>lambertw(-1, $\frac{3 \cdot \pi}{2}$) = $-1.5 \cdot \pi \cdot i$</code>
<code>lambertw(1, 0) = $-\infty$</code>

Precision := Approximate

$$\text{lambertw}(1, 3 - 2 \cdot i) = -0.1273564243 + 4.093282894 \cdot i$$

$$\text{lambertw}(1, 3 + 2 \cdot i) = -0.3742879384 + 5.22893329 \cdot i$$

$$\text{lambertw}\left(0, \frac{i}{2}\right) = 0.1625996482 + 0.3926285739 \cdot i$$

$$\text{lambertw}(0, 5 \cdot i) = -0.2233736303 + 6.24744614 \cdot i$$

$$\text{lambertw}(0, -5 \cdot i) = -0.2233736303 - 6.24744614 \cdot i$$

$$\text{lambertw}(0, -1.5) = -0.03278373591 + 1.549643823 \cdot i$$

$$\text{lambertw}(0, 1.5) = 0.7258613698$$

$$\text{lambertw}(-1, -0.2) = -0.2591710984$$

$$\text{lambertw}(-1, 3 \cdot i) = 11.70471152 + 0.3660158906 \cdot i$$

$$\text{lambertw}(2, 3 \cdot i) = -1.429822035 + 12.45204504 \cdot i$$

$$\text{lambertw}(-1, 2 - 3 \cdot i) = -0.4462717121 - 5.615883357 \cdot i$$

$$\text{lambertw}(-1, 2 + 3 \cdot i)$$

Memory exhausted, but:

$$\text{lambertw}(2, 2 + 3 \cdot i) = -1.197260107 + 11.87791011 \cdot i$$

$$\text{lambertw}(-2, 2 + 3 \cdot i) = -1.016359781 - 9.910584876 \cdot i$$

$$\text{lambertw}(-1, -2 + i) = 0.4238068940 + 1.400916834 \cdot i$$

<code>kit_ets_mb\lambertw(1,3-2*i)</code>	<code>-0.127356+4.09328*i</code>
<code>kit_ets_mb\lambertw(1,3+2*i)</code>	<code>-0.374288+5.22893*i</code>
<code>kit_ets_mb\lambertw(0,i/2)</code>	<code>0.1626+0.392629*i</code>
<code>kit_ets_mb\lambertw(0,5*i)</code>	<code>1.19805+0.917331*i</code>
<code>kit_ets_mb\lambertw(0,-5*i)</code>	<code>1.19805-0.917331*i</code>
<code>kit_ets_mb\lambertw(0,-1.5)</code>	<code>-0.032784+1.54964*i</code>
<code>kit_ets_mb\lambertw(0,1.5)</code>	<code>0.725861</code>
<code>kit_ets_mb\lambertw(-1,2-3*i)</code>	<code>-0.446272-5.61588*i</code>
<code>kit_ets_mb\lambertw(-1,2+3*i)</code>	<code>-0.031583-3.72111*i</code>
<code>kit_ets_mb\lambertw(-1,3*i)</code>	<code>-0.041932-3.12819*i</code>
<code>kit_ets_mb\lambertw(-1,-0.2)</code>	<code>-2.54264</code>

The real and some complex solutions of Problem 3c):

$$\#41: -\frac{4}{\text{LOG}(3)} \cdot \left[\text{lambertw}\left(0, \frac{\text{LOG}(3)}{4}\right), \text{lambertw}\left(0, -\frac{\text{LOG}(3)}{4}\right), \text{lambertw}\left(-1, -\frac{\text{LOG}(3)}{4}\right) \right]$$

$$\#42: [-0.8022464315, 1.516776412, 7.174755827]$$

$$\#43: -\frac{4}{\text{LN}(3)} \cdot \text{lambertw}\left(0, \frac{i \cdot \text{LN}(3)}{4}\right) = -0.2318785967 - 0.9091964141 \cdot i$$

$$\#44: -\frac{4}{\text{LN}(3)} \cdot \text{lambertw}\left(0, -\frac{i \cdot \text{LN}(3)}{4}\right) = -0.2318785967 + 0.9091964141 \cdot i$$

$$\#45: -\frac{4}{\text{LN}(3)} \cdot \text{lambertw}\left(-1, \frac{i \cdot \text{LN}(3)}{4}\right) = 11.57399382 - 21.04616178 \cdot i$$

$$\#46: -\frac{4}{\text{LN}(3)} \cdot \text{lambertw}\left(-1, -\frac{i \cdot \text{LN}(3)}{4}\right) = 11.57399382 + 21.04616178 \cdot i$$

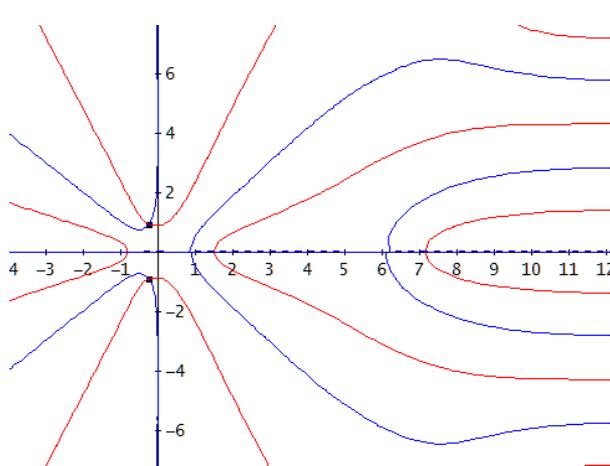
$$\#47: -\frac{4}{\text{LN}(3)} \cdot \text{lambertw}\left(2, \frac{i \cdot \text{LN}(3)}{4}\right) = 14.00201516 - 44.64712529 \cdot i$$

$$\#48: -\frac{4}{\text{LN}(3)} \cdot \text{lambertw}\left(-2, \frac{i \cdot \text{LN}(3)}{4}\right) = 12.98784050 + 32.94807188 \cdot i$$

$$\text{RE}((x + i \cdot y)^4) = \text{RE}(3^{x + i \cdot y})$$

$$\text{IM}((x + i \cdot y)^4) = \text{IM}(3^{x + i \cdot y})$$

$$\begin{bmatrix} -0.2318785967 & -0.9091964141 \\ -0.2318785967 & 0.9091964141 \\ 11.57399382 & -21.04616178 \\ 11.57399382 & 21.04616178 \\ 14.00201516 & -44.64712529 \\ 12.9878405 & 32.94807188 \end{bmatrix}$$



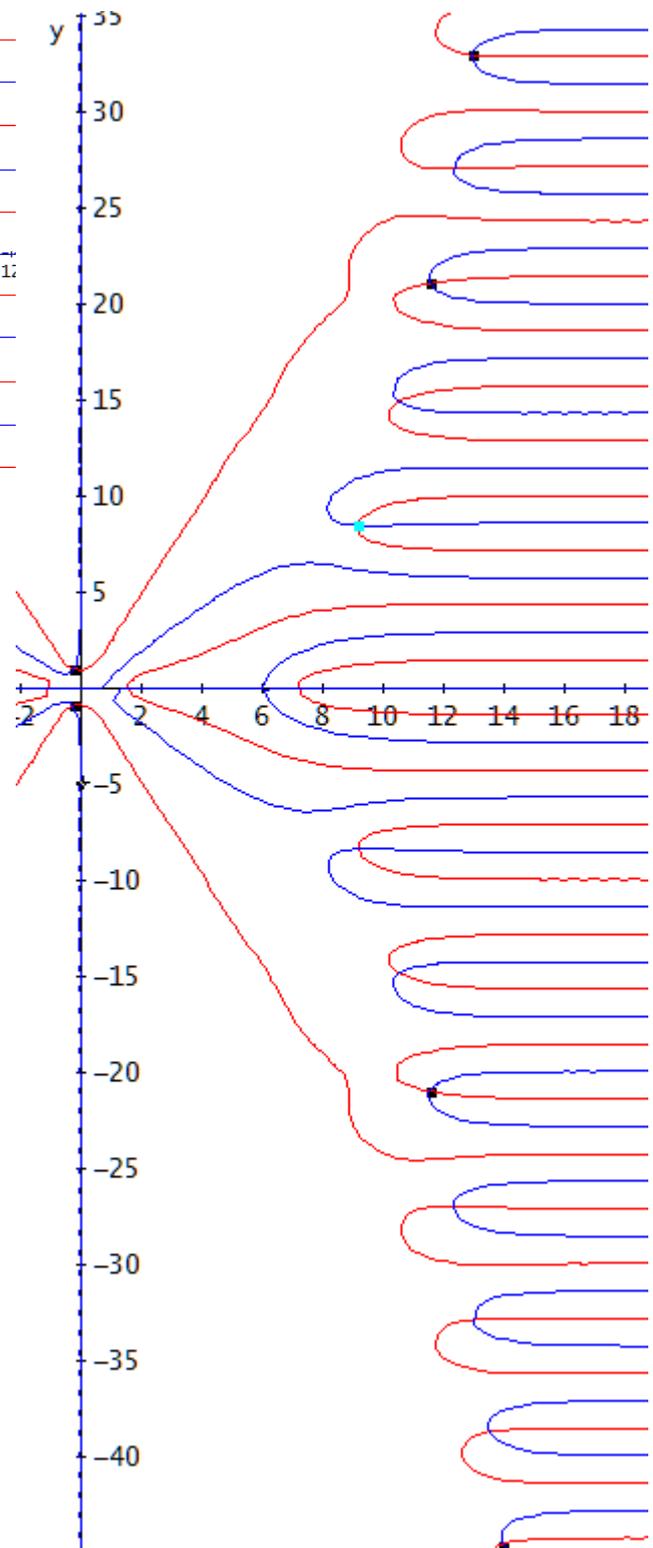
These are the points of my solutions. For #45 from above you get another complex number. I plotted it in light blue – and not surprisingly it is also an intersection point.

$$\frac{-4}{\ln(3)} \cdot \text{kat_ets_mbVambertw}\left(-1, \frac{i \cdot \ln(3)}{4}\right)$$

9.18461+8.42098 · i

Finally, I added a slightly changed version of your program (in my DERIVE version) to overcome the problem with hanging up during calculation.

I add a third parameter which are trials for initial values for the Newton method. As you can see in the last line, I can discover your complex number, too. The initial values could be obtained from a graph.



$$-\frac{4}{\text{LN}(3)} \cdot \text{lambertw2}\left(-1, \frac{i \cdot \text{LN}(3)}{4}, [-2, -2]\right) = 9.184614960 + 8.420975690 \cdot i$$

From Michel to Josef

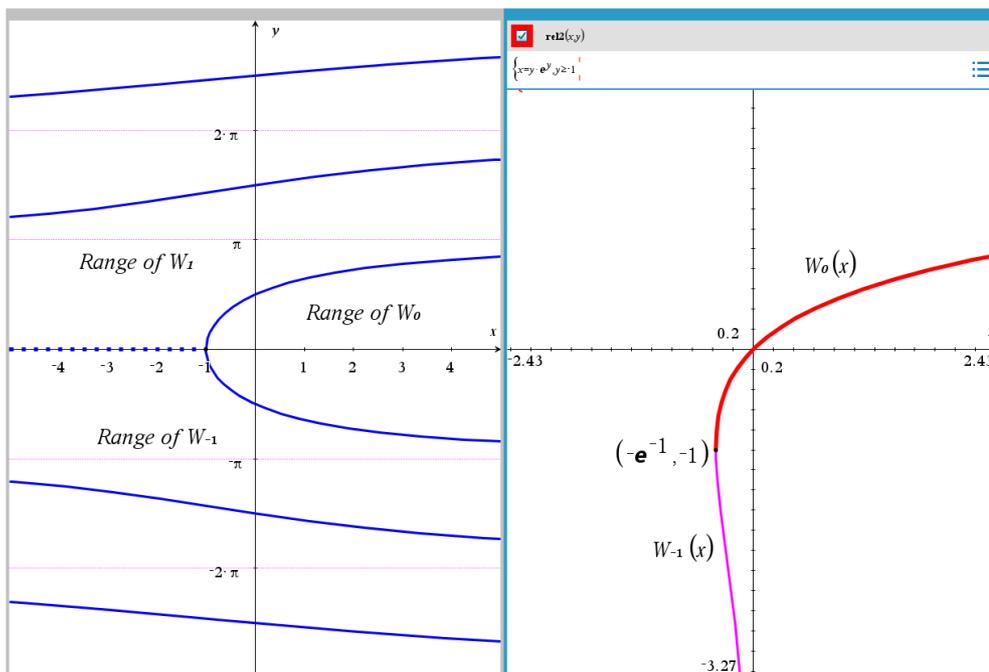
When Jeffrey, Corless, Gonnet, Knuth and Hare programmed LambertW, they have decided that the branch cut will "follow" what is happening with the Log function. So if you set the imaginary part to be 0, you get the ranges of every branches "k" of LambertW. See the file Branches.

I also learn many interesting things talking and exchanging emails with you. We form a good team ! Today I have sent an email to the guy in Warsaw for proposing the Education session next summer. The website should be online soon. If you think of sending a proposal, I will be happy to do the Latex for you. Take care and Happy Holidays to you and Noor,

See the attachment in order to know how I was able to produce figure 11 using Nspire. Because there is still no real 2D implicit plotting available --only for polynomial relation or curves given by $x = g(y)$. And neither equation of the real part nor the imaginary part can be solved for x or for y .

Can Santa bring a real 2D implicit plotter for Christmas ? Derive has it since almost 30 years ! Is it because TI develops the handheld with the software and it won't be fast enough on the handheld ? Take care,

$z = e^z _{z=x+iy}$	$(x \cdot \cos(y) - y \cdot \sin(y)) \cdot e^x + (x \cdot \sin(y) + y \cdot \cos(y)) \cdot e^x \cdot i$
©If we set the imaginary part to 0, then we get this because e^x is never 0:	
$\text{zeros}(x \cdot \sin(y) + y \cdot \cos(y), x)$	$\left\{ \begin{array}{l} -y \\ \tan(y) \end{array} \right\}$
⚠ ©So the curve $x = -y \cdot \cot(y)$ is plotted (in relation mode or in parametric mode). See next page	



“Implicit Plot” wit TI-Nspire (Michel’s trick):

```

3^{x+i} \cdot y_{-(x+i)} y^4
3^x \cdot \cos(\ln(3) \cdot y) - x^4 + 6 \cdot x^2 \cdot y^2 - y^4 + (3^x \cdot \sin(\ln(3) \cdot y) - 4 \cdot x \cdot (x^2 - y^2)) \cdot y \cdot i
real(3^x \cdot \cos(\ln(3) \cdot y) - x^4 + 6 \cdot x^2 \cdot y^2 - y^4 + 3^x \cdot \sin(\ln(3) \cdot y) - 4 \cdot x \cdot (x^2 - y^2)) \cdot y
3^x \cdot (\cos(\ln(3) \cdot y) + \sin(\ln(3) \cdot y)) - x^4 - 4 \cdot x^3 \cdot y + 6 \cdot x^2 \cdot y^2 + 4 \cdot x \cdot y^3 - y^4
imag(3^x \cdot \cos(\ln(3) \cdot y) - x^4 + 6 \cdot x^2 \cdot y^2 - y^4 + (3^x \cdot \sin(\ln(3) \cdot y) - 4 \cdot x \cdot (x^2 - y^2)) \cdot y \cdot i)
3^x \cdot \sin(\ln(3) \cdot y) - 4 \cdot x \cdot (x^2 - y^2) \cdot y

t1:=taylor(3^x,x,20)
1+ln(3) \cdot x + \frac{(\ln(3))^2 \cdot x^2}{2} + \frac{(\ln(3))^3 \cdot x^3}{6} + \frac{(\ln(3))^4 \cdot x^4}{24} + \frac{(\ln(3))^5 \cdot x^5}{120} + \frac{(\ln(3))^6 \cdot x^6}{720} + \frac{(\ln(3))^7 \cdot x^7}{5040} + \frac{(\ln(3))^8 \cdot x^8}{40320} + \frac{(\ln(3))^9 \cdot x^9}{362880} + \frac{(\ln(3))^{10} \cdot x^{10}}{3628800} + \frac{(\ln(3))^{11} \cdot x^{11}}{39916800}

t2:=taylor(cos(ln(3) \cdot y),y,20)
1 - \frac{(\ln(3))^2 \cdot y^2}{2} + \frac{(\ln(3))^4 \cdot y^4}{24} - \frac{(\ln(3))^6 \cdot y^6}{720} + \frac{(\ln(3))^8 \cdot y^8}{40320} - \frac{(\ln(3))^{10} \cdot y^{10}}{3628800} + \frac{(\ln(3))^{12} \cdot y^{12}}{479001600} - \frac{(\ln(3))^{14} \cdot y^{14}}{87178291200} + \frac{(\ln(3))^{16} \cdot y^{16}}{20922789888000} - \frac{(\ln(3))^{18} \cdot y^{18}}{6402373705728000} + \dots

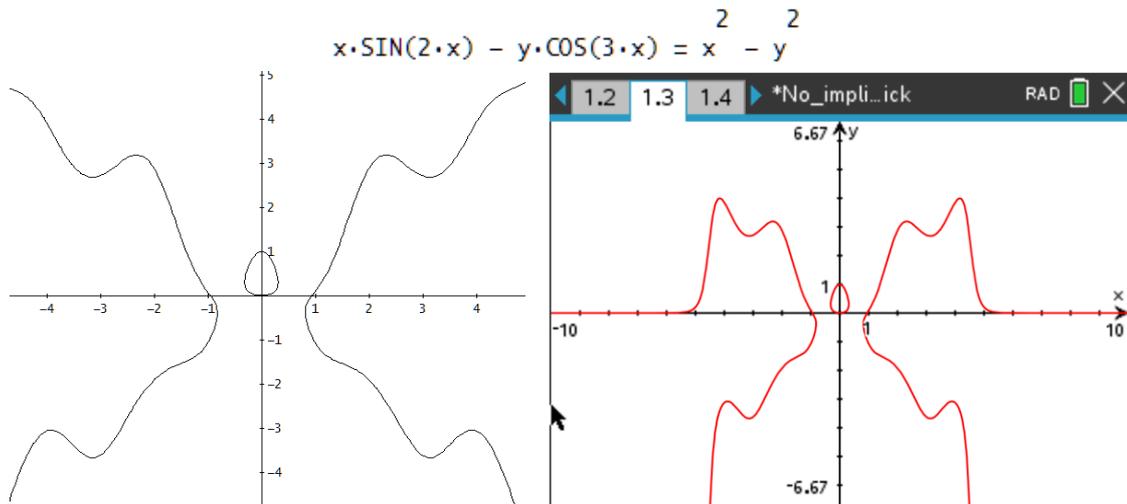
t3:=taylor(sin(ln(3) \cdot y),y,20)
ln(3) \cdot y - \frac{(\ln(3))^3 \cdot y^3}{6} + \frac{(\ln(3))^5 \cdot y^5}{120} - \frac{(\ln(3))^7 \cdot y^7}{5040} + \frac{(\ln(3))^9 \cdot y^9}{362880} - \frac{(\ln(3))^{11} \cdot y^{11}}{39916800} + \frac{(\ln(3))^{13} \cdot y^{13}}{6227020800} - \frac{(\ln(3))^{15} \cdot y^{15}}{1307674368000} + \frac{(\ln(3))^{17} \cdot y^{17}}{355687428096000} - \frac{(\ln(3))^{19} \cdot y^{19}}{1216451004000}

t1 \cdot t2 - x^4 + 6 \cdot x^2 \cdot y^2 - y^4
\frac{(\ln(3))^{20} \cdot x^{20} \cdot ((\ln(3))^{20} \cdot y^{20} - 380 \cdot (\ln(3))^{18} \cdot y^{18} + 116280 \cdot (\ln(3))^{16} \cdot y^{16} - 27907200 \cdot (\ln(3))^{14} \cdot y^{14} + 5079110400 \cdot (\ln(3))^{12} \cdot y^{12} - 670442572800 \cdot (\ln(3))^{10} \cdot y^{10} + 591901 \cdot (\ln(3))^{8} \cdot y^8 - 342 \cdot (\ln(3))^{6} \cdot y^6 + 93024 \cdot (\ln(3))^{4} \cdot y^4 - 19535040 \cdot (\ln(3))^{2} \cdot y^2 + 3047466240 \cdot (\ln(3))^{10} \cdot y^{10} - 335221286400 \cdot (\ln(3))^{12} \cdot y^{12} + \dots}{591901}

t1 \cdot t3 - 4 \cdot x \cdot (x^2 - y^2) \cdot y
-\frac{(\ln(3))^{21} \cdot x^{20} \cdot ((\ln(3))^{18} \cdot y^{18} - 342 \cdot (\ln(3))^{16} \cdot y^{16} + 93024 \cdot (\ln(3))^{14} \cdot y^{14} - 19535040 \cdot (\ln(3))^{12} \cdot y^{12} + 3047466240 \cdot (\ln(3))^{10} \cdot y^{10} - 335221286400 \cdot (\ln(3))^{12} \cdot y^{12} + \dots)}{591901}
    
```

Then plot the two relations: $rel1(x,y) = t1 \cdot t2 - x^4 + 6x^2y^2 - y^4 = 0$ and $rel2(x,y) = t1 \cdot t3 - 4x(x^2 - y^2)y = 0$.

I tried another example: Compare the DERIVE implicit plot with the approximated Nspire’s one:



```

t5:=x \cdot taylor(sin(2 \cdot x),x,30)
2 \cdot x^2 \cdot (8 \cdot x^{28} - 1624 \cdot x^{26} + 285012 \cdot x^{24} - 42751800 \cdot x^{22} + 5408102700 \cdot x^{20} - 567850783500 \cdot x^{18} + 48551241989250 \cdot x^{16} - 3301484455269000 \cdot x^{14} + \dots)

t6:=y \cdot taylor(cos(3 \cdot x),x,30)
(1 - \frac{9 \cdot x^2}{2} + \frac{27 \cdot x^4}{8} - \frac{81 \cdot x^6}{80} + \frac{729 \cdot x^8}{4480} - \frac{729 \cdot x^{10}}{44800} + \frac{2187 \cdot x^{12}}{1971200} - \frac{19683 \cdot x^{14}}{358758400} + \frac{59049 \cdot x^{16}}{28700672000} - \frac{59049 \cdot x^{18}}{975822848000} + \frac{531441 \cdot x^{20}}{370812682240000} - \frac{1594323 \cdot x^{22}}{571051530649600} + \dots)

t7:=t5-t6-(x^2-y^2)
\frac{x^{30} \cdot (68630377364883 \cdot y + 5368709120) \cdot x^{28} \cdot (22876792454961 \cdot y + 3758096384) + x^{26} \cdot (2541865828329 \cdot y + 872415232) \cdot x^{24} \cdot (94143178827 \cdot y + 88417619937397019545436160000000) - 304888344611713860501504000000 - 403291461126605635584000000 - 2068161339110798}{591901}
    
```

Problems 4

Solving systems of nonlinear equations by the fixed-point method or by Newton's method

- a) Find the real solutions of the following system by the fixed-point method (use implicit 2D- or 3D-graphs to obtain an initial point):

$$\begin{cases} x-1 = \frac{1}{1+(x+y)^4} \\ y-3 = \frac{1}{1+(x-y)^4} \end{cases}$$

- b) Find the real solutions of the following system using Newton's method (use implicit 2D graphs to get an initial point):

$$\begin{cases} x^3 + y^3 = 10 \\ x y^2 - x^2 y = 2 \end{cases}$$

Note that the latter system is polynomial and can be solved exactly. If you can, take a look at a Gröbner basis given by a symbolic system to see that a polynomial equation of degree 9 appears in y , but reducible to an equation of degree 3. Since the Cardan formulas are known, one can understand why an exact solution is possible but not practical.

- c) Find the real solutions of the following system using Newton's method:

$$\begin{cases} 3x y - 2x^2 + 4 \sin y + 6 = 0 \\ 3x^2 - 2x y^2 + 3 \cos x + 4 = 0 \end{cases}$$

- d) Find the real solutions of the following system using Newton's method:

$$\begin{cases} x^2 - x y + 2y - 10 = 0 \\ x^3 y^2 - 2 = 0 \end{cases}$$

- e) Find the real solutions of the following system using Newton's method:

$$\begin{cases} 12 + 3x + 8y + 12x y + 9x^2 - 12y^2 = 0 \\ 5 + 12x + 6y - 9x y - 2x^2 - 11y^2 = 0 \end{cases}$$

- f) Consider the following system of equations $\begin{cases} x^2 + y^2 = 16 \\ x y = 1 \end{cases}$.

- i) Give a simple argument to show that this system has 4 real solutions.
- ii) Find a Gröbner base of this system.
- iii) Show that if you have found a solution, then you have the other three ones, too.
- iv) Find one of the four solutions, exactly. Reconfirm by Newton with two variables.

- g) We are looking for a complex solution to the equation $x^2 = 2^x$. We then substitute x by $x + i y$ to obtain two equations with two unknowns (the real part, the imaginary part). Show that using Newton's method in two variables and choosing as starting point the couple $(3.1, 10.4)$ we get the real solution $x = 2$. Show that if the starting point is rather the couple $(7.1, 12.4)$ then there we find indeed a complex solution.

Problem 4a)

Find the real solutions of the following system by the fixed-point method (use implicit 2D or 3D graphs to get an initial point):

$$\begin{cases} x-1 = \frac{1}{1+(x+y)^4} \\ y-3 = \frac{1}{1+(x-y)^4} \end{cases}$$

To plot each of the two curves, an implicit 2D-plotter is required but fortunately (if you use Nspire), the equations are equivalent to polynomial equations and here are each of these two curves in the same window: an intersection point seems to be around (1, 3):

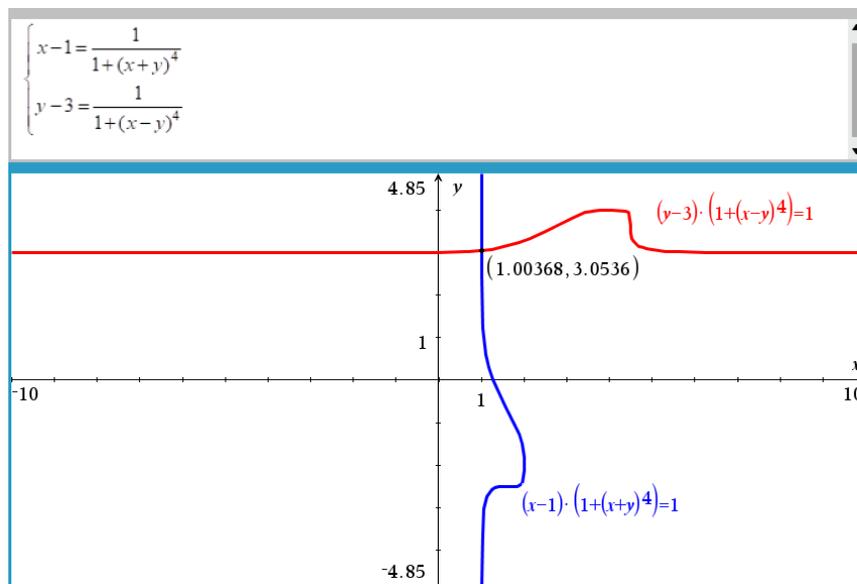


Figure 3

We will apply the fixed-point method without checking if the absolute value of the determinant of the Jacobian matrix of the vector field $\mathbf{F}: \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R} \times \mathbb{R}$ defined by

$$\mathbf{F}(x, y) = \left[1 + \frac{1}{1+(x+y)^4}, 3 + \frac{1}{1+(x-y)^4} \right]$$

is less than one in a neighborhood of the point (1, 3), which is probably the case. The reader can verify that the *Maple* command "*fsolve*" finds the solution

$$\{x = 1.003676746, y = 3.053595699\}.$$

In Nspire -- we decided to display more decimals in Figure 4 -- the fixed-point method confirms this answer:

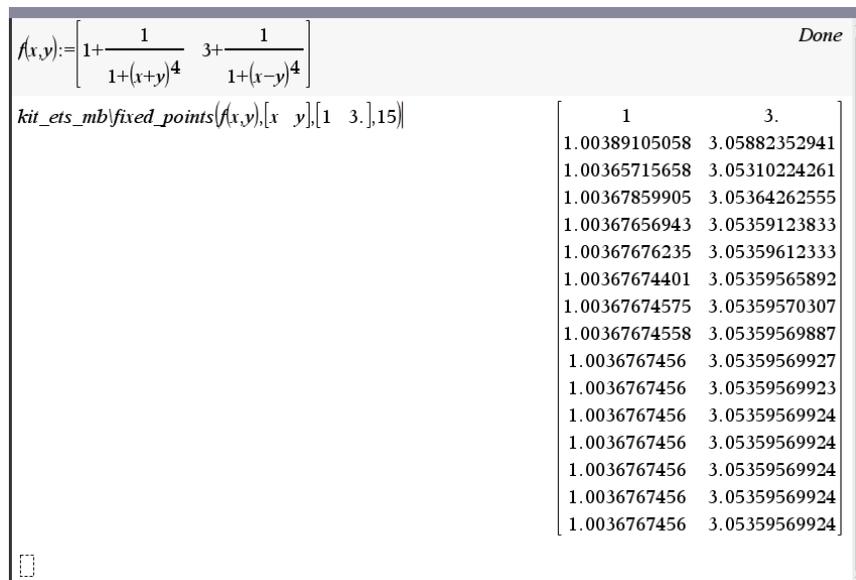


Figure 4

However, the reader can check that the Nspire "solve" command gives several answers, all to be rejected: this is probably due to the fact that the precision of the calculation is limited to 14 digits (even on the software version).

Remark: it is however easy to verify that the continuous vector field $\mathbf{F}(x,y)$ sends a rectangle into itself and that, on this rectangle, the absolute value of the determinant of its Jacobian matrix is indeed increased by a constant K strictly less than one. In fact, let's take the rectangle (square)

$R = \{(x, y) \in \mathbb{R}^2 \mid 2 \leq x \leq 3, 3 \leq y \leq 4\}$. If we take a point (x,y) of this rectangle, then since

$0 < \frac{1}{1+(x \pm y)^4} < 1$, it is clear that \mathbf{F} sends R into itself. By using a calculator to calculate the determinant

of the Jacobian matrix of \mathbf{F} and drawing the graph of the resulting scalar field, we can convince ourselves that the surface is well included between the two horizontal planes $z = 0$ and $z = 1$. Thus, whatever the starting point (a, b) chosen in R , we would have had convergence! In particular, starting from the "corner" of the rectangle at point $(2, 4)$ we receive:

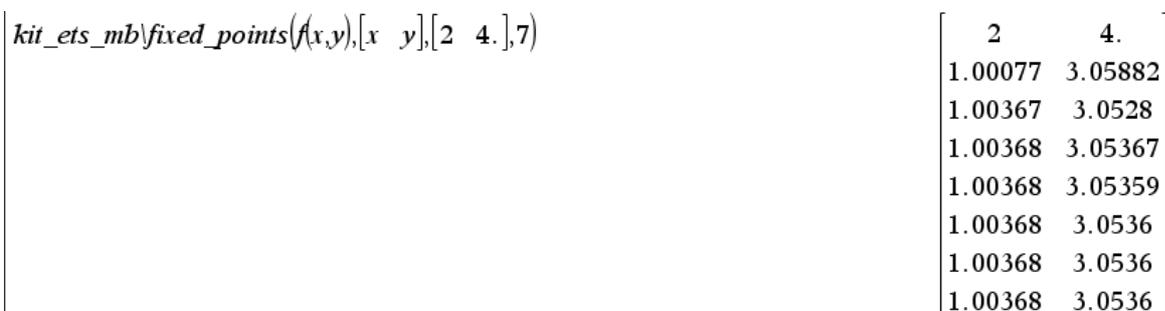


Figure 5

One "fixed-point" calculation performed with DERIVE is given on the next page.

$$\text{FIXED_POINT} \left(\left[1 + \frac{1}{1 + (x + y)^4}, 3 + \frac{1}{1 + (x - y)^4} \right], [x, y], [2, 4], 5 \right)$$

$$\begin{bmatrix} 2 & 4 \\ 1.000771010 & 3.058823529 \\ 1.003668371 & 3.052797921 \\ 1.003679658 & 3.053673900 \\ 1.003676452 & 3.053588248 \\ 1.003676773 & 3.053596407 \end{bmatrix}$$

Problem 4b)

Find the real solutions of the following system using Newton's method (use implicit 2D graphs to get a starting point):

$$\begin{cases} x^3 + y^3 = 10 \\ x y^2 - x^2 y = 2 \end{cases}$$

Note that the latter system is polynomial and can be solved exactly. A Gröbner basis of this system is provided by the *Derive* software here. You can choose the lexicographical order (x, y) or (y, x) : line #3 of figure 6 shows that the system is equivalent to another one whose first expression is a polynomial in "y" that can be reduced to degree 3, thus solved symbolically by the Cardano formula. The same is true in line #5 for the polynomial in "x". That's why this system is symbolically solvable (but it's not very practical!)

```
#1: syst := [x^3 + y^3 - 10, x*y^2 - x^2*y - 2]
#2: GROEBNER_BASIS(syst, [x, y])
#3: [y^9 - 18*y^6 + 80*y^3 + 4, 10*x + y^7 - 16*y^4 + 58*y]
#4: GROEBNER_BASIS(syst, [y, x])
#5: [x^9 - 12*x^6 + 20*x^3 - 4, 10*y - x^7 + 14*x^4 - 38*x]
```

Figure 6

Someone who wanted to solve the equation $x^9 - 12x^6 + 20x^3 - 4 = 0$ would be able to do so, since by posing the latter equation we transform it into $z^3 - 12z^2 + 20z - 4 = 0$. And we know how to solve a third degree polynomial equation symbolically. The three values we would find for z would be real and positive and we would take the cube root of each of them to obtain the values of "x". Each of these values would then be used to find the corresponding value of "y" since line #5 of Figure 6 then indicates that

$$y = \frac{x^7 - 14x^4 + 38x}{10}.$$

Nevertheless, the answers obtained would be impractical ... and one would then have to approximate them in decimal form to use them. Newton's method with two variables will give us the same decimals!

To use Newton's method with two variables, we will plot each of the two curves on the same axes in order to look for intersections. Graphical analysis and an intersection tool already find three solutions:

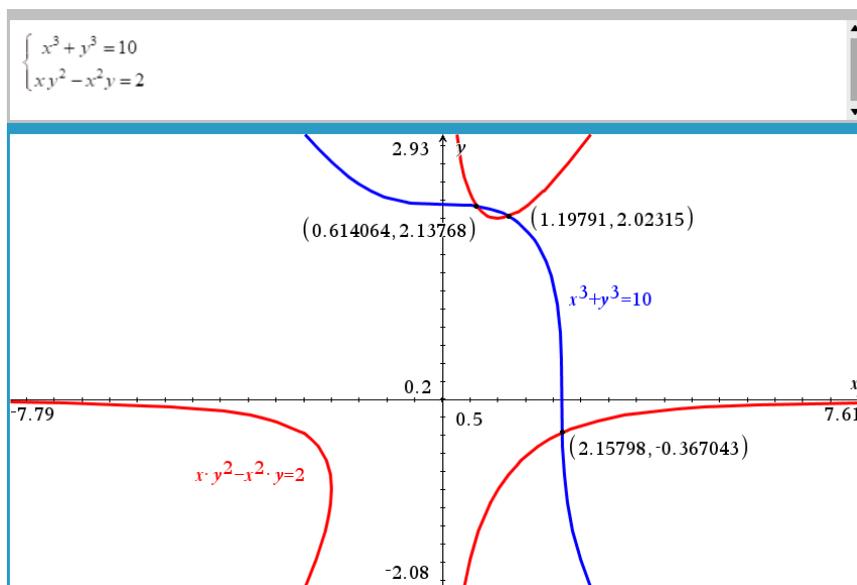


Figure 7

Let's confirm these three intersections using Newton's method with two variables. With a given starting point, a convergent numerical method can only find one solution at a time. Let's make a function that uses the "Newtons2" function from the kit_ets_mb.tns library and that depends on the starting point (a,b) .

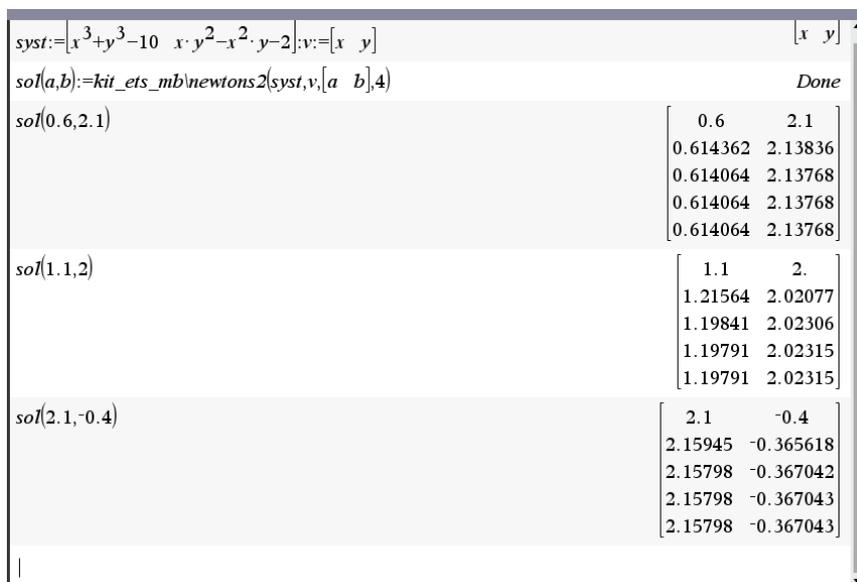


Figure 8

The convergence is very fast: only 4 iterations were sufficient to observe a convergence.

Next page shows how we can solve the equation with DERIVE. The last expression is pretty huge. You can see only a part of the exact real solutions.

NEWTONS($[x^3 + y^3 - 10, x^2 \cdot y - x \cdot y^2 - 2]$, $[x, y]$, $[0.6, 2.1]$, 4)

$$\begin{bmatrix} 0.6 & 2.1 \\ 0.6143622722 & 2.138358937 \\ 0.6140641341 & 2.137676392 \\ 0.6140640207 & 2.137676171 \\ 0.6140640207 & 2.137676171 \end{bmatrix}$$

SOLUTIONS($[x^3 + y^3 = 10, x^2 \cdot y - x \cdot y^2 = 2]$, $[x, y]$)

$$\begin{bmatrix} 2.157979959 & -0.3670432295 \\ 0.6140640208 & 2.137676171 \\ 1.1979139 & 2.023147422 \\ -1.078989979 + 1.868865465 \cdot i & 0.1835216147 - 0.317868761 \cdot i \\ -1.078989979 - 1.868865465 \cdot i & 0.1835216147 + 0.317868761 \cdot i \\ -0.5989569504 + 1.037423869 \cdot i & -1.011573711 + 1.752097063 \cdot i \\ -0.5989569504 - 1.037423869 \cdot i & -1.011573711 - 1.752097063 \cdot i \\ -0.3070320104 + 0.5317950415 \cdot i & -1.068838085 + 1.851281869 \cdot i \\ -0.3070320104 - 0.5317950415 \cdot i & -1.068838085 - 1.851281869 \cdot i \end{bmatrix}$$

SOLUTIONS($[x^3 + y^3 = 10, x^2 \cdot y - x \cdot y^2 = 2]$, $[x, y]$, Rea1)

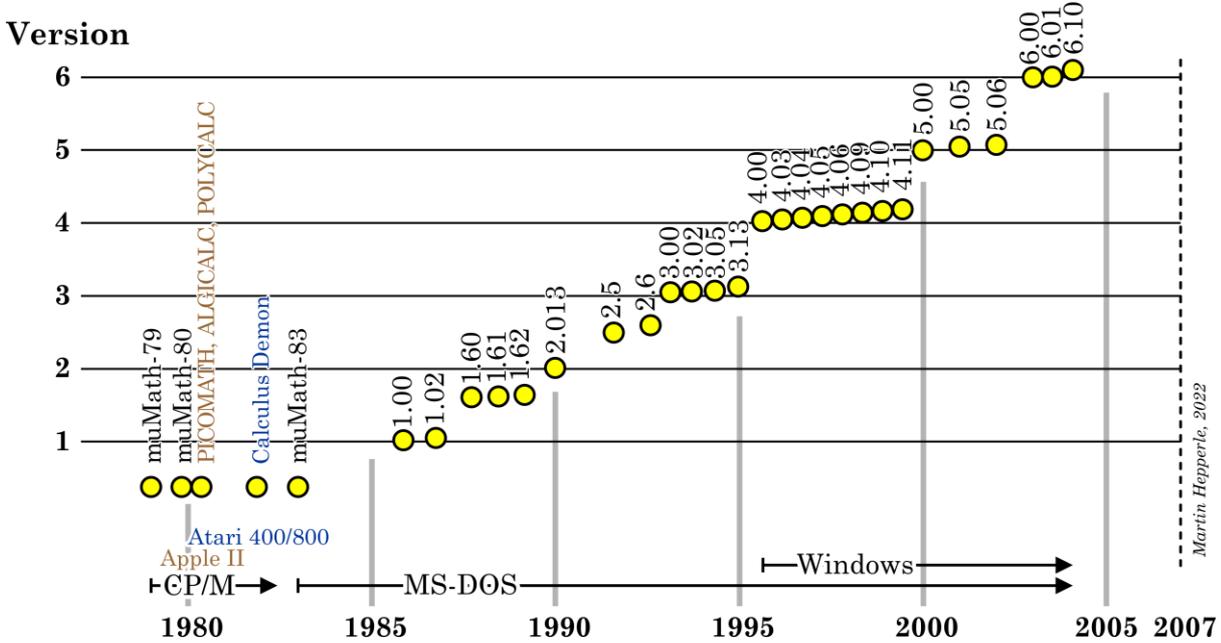
$$\begin{bmatrix} 2.157979959 & -0.3670432295 \\ 0.6140640208 & 2.137676171 \\ 1.1979139 & 2.023147422 \end{bmatrix}$$

$$\begin{bmatrix} \frac{2 \cdot 2^{1/3} \cdot 3^{5/6} \cdot \left(2 \cdot \sqrt{7} \cdot \cos\left(\frac{\text{ATAN}\left(\frac{5 \cdot \sqrt{111}}{117}\right)}{3}\right) - 3 \cdot \sqrt{3}\right)^{7/3}}{45} + \frac{16 \cdot 6^{1/3} \cdot \left(2 \cdot \sqrt{7} \cdot \cos\left(\frac{\text{ATAN}\left(\frac{5 \cdot \sqrt{111}}{117}\right)}{3}\right) - 3 \cdot \sqrt{3}\right)^{4/3}}{15} + \dots \\ - \frac{2 \cdot 2^{1/3} \cdot 3^{5/6} \cdot \left(2 \cdot \sqrt{7} \cdot \sin\left(\frac{\text{ACOT}\left(-\frac{5 \cdot \sqrt{111}}{117}\right)}{3}\right) + 3 \cdot \sqrt{3}\right)^{7/3}}{45} + \frac{16 \cdot 6^{1/3} \cdot \left(2 \cdot \sqrt{7} \cdot \sin\left(\frac{\text{ACOT}\left(-\frac{5 \cdot \sqrt{111}}{117}\right)}{3}\right) + 3 \cdot \sqrt{3}\right)^{4/3}}{15} - \dots \\ - \frac{2 \cdot 2^{1/3} \cdot 3^{5/6} \cdot \left(2 \cdot \sqrt{7} \cdot \cos\left(\frac{\text{ATAN}\left(\frac{5 \cdot \sqrt{111}}{117}\right)}{3}\right) + \frac{\pi}{3}\right) + 3 \cdot \sqrt{3}}{45} + \frac{16 \cdot 6^{1/3} \cdot \left(2 \cdot \sqrt{7} \cdot \cos\left(\frac{\text{ATAN}\left(\frac{5 \cdot \sqrt{111}}{117}\right)}{3}\right) + \frac{\pi}{3}\right) + 3 \cdot \sqrt{3}}{15} \\ - \frac{2 \cdot 2^{1/3} \cdot 3^{5/6} \cdot \left(2 \cdot \sqrt{7} \cdot \cos\left(\frac{\text{ATAN}\left(\frac{5 \cdot \sqrt{111}}{117}\right)}{3}\right) - 3 \cdot \sqrt{3}\right)^{1/3}}{3} \end{bmatrix}$$

To be continued

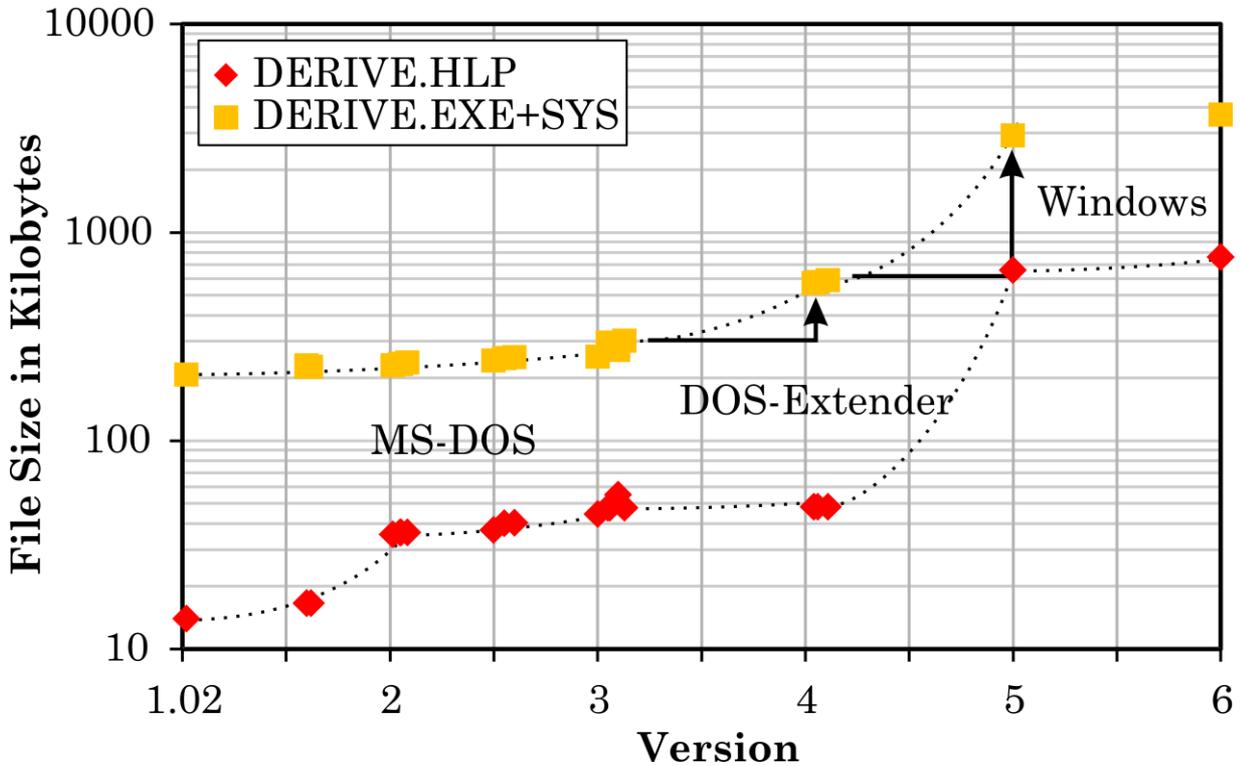
Development of DERIVE - versions

I helped Dr Hepperle accomplishing his collection of the DOS versions of DERIVE. He used and still uses DERIVE on his HP-Portable, HP-150 oder HP-200LX Palmtop for demonstration purposes. He sent two time lists showing the development of the DERIVE versions:



Version History - Soft Warehouse Derive

Derive's Growth



There are a few questions and problems which appear regularly in my mail box. One of them is connected with every new release of WINDOWS. "What to do when reinstalling DERVE in an updated WINDOWS environment fails?" Marcelo from Argentina complained a long time. But fortunately we could find the final solution in our DUG Community. Thanks to all of you who helped.

DERIVE and WINDOWS11

Marcelo from Argentina wrote:

Dear Josef,

I hope you are fine. I was unable to install Derive on Windows 11. Did anybody solve the problem?

I had no problems with Windows XP and later with Windows 7. However, the setup does not work on Windows 11.

Best regards,

Marcelo

As I don't work with WIN11, I asked in our community and I received many answers. Once more, thanks to all of you who sent answers and advice.

Lars Erup wrote:

I'm using Derive for Windows v 5.00 on Windows 11. It seems to work fine; I'm not doing particularly sophisticated things but haven't noticed anything untoward.

Cheers

Lars

Danny Ross Lunsford wrote:

There is no WOW (Windows on Windows) capability on Windows 11 to run 16-bit applications at all. Your best bet is to run a virtual machine running an earlier 32-bit version of Windows (any from 2000 to 10 will do) under say Virtual Box and install Derive in that. Because Derive does not have sophisticated graphics capabilities, you will lose nothing from the VM's lesser video performance and you will not even notice that it is running in a VM after a while.

I have been running 32-bit Derive 6.10 on Windows 10 for a long time. Haven't tried Windows 11 but I think it would work the same way. Windows 11 still allows 32-bit software. The same song and dance that enables winhlp32 on Windows 10 should also work.

George Cook wrote:

Hello Josef

My copy of Derive (version 6.10) installed on Windows 11 without any issues.

Although I have not used it a lot, it seems to operate normally.

These days, I spend most of my time working with Python.

I started my Python journey about 6 months ago and I am very excited about it.

This is my first experience with programming and there is much to learn.

I recommend learning Python, the payoff is well worth the effort.

One more mail from George (accompanied by a couple of attachments):

Hello Josef:

At one time I was a member of DUG and you now have my Email address, so please renew my membership.

Yes, I use both N-Spire and Python on my PC as well as Derive.

I am using PyCharm (community edition) as my Python IDE.

I wish I had gotten interested in Python years ago, it is really an important skill to have for doing Mathematics.

By the way, I got an Email from Francisco asking if I could solve the Derive HELP problem in Windows 11.

My solution was to copy use my old computer (windows 7) and copy the HELP files back into Derive's text editor.

I copy and paste the HELP examples back into my new Derive document for guidance.

I have attached these files for you and hope that you find them useful.

Please let me know if this idea works for you.

George

George's files can be found in the folder `COOK` in `MTH128.zip`. George did a tremendous job, many thanks. See the first lines of one of his files (Derive Help.docx):

Utility File Functions

The following is an alphabetical list of the functions defined in the utility files in Math directory distributed with Derive (see the Utility File Library). These functions are automatically load on demand.

ADJOINT(A) adjoint of square matrix A

AI_SERIES(z,m) m+1 terms of series approximation for Airy function $A_i(z)$

ALMOST_LIN(r,b,p,q,x,y,x0,y0) implicit solution of $r(x,y)y' + p(x)b(y) = q(x)$ if almost linear

There are two WORD and 15 DERIVE dfw-files in the folder.

See the beginning of his file Built-in Functions and Constants page2.dfw:

Expression Type Functions

The functions in this topic are primarily used when writing Derive function definitions to determine the data type of expressions. By convention, the function names includes a ? to emphasize that they are predicates that return a truth-value (i.e. true or false).

INTEGER?(u) simplifies to true if u is an integer; otherwise it simplifies to false. For example, 5, 0, -1, and 3 are integers. Note that, the expressions FLOOR(x) and SIGN(x) are not integers, although their data type is integer. Use INTEGER-TYPE? to test for such expressions.

RATIONAL?(u) simplifies to true if u is rational number (including integers); otherwise it simplifies to false. For example, 2/3, -7/2, 5, and 0 are rational numbers.

EVEN?(u) simplifies to true if u is an even number (including fractions whose numerator or denominator is even); otherwise it simplifies to false. For example, 4, 0, -6, 2/3, and 3/2 are even numbers.

ODD?(u) simplifies to true if u is an odd number (including fractions whose numerator and denominator is odd); otherwise it simplifies to false. For example, 1, -7, and 3/5 are odd numbers.

NUMBER?(u) simplifies to true if u is a real or complex number; otherwise it simplifies to false. For example, 3, -2/3, $2 \cdot i$, and $3 - 5 \cdot i$ are numbers. Note that, the expressions SIN(x) and x^2 are not numbers, although their data type is number. Use REAL_TYPE? and NUMBER-TYPE? to test for such expressions.

VARIABLE?(u) simplifies to true if u is a variable; otherwise it simplifies to false. For example, x, a, and Real are variables.

STRING?(u) simplifies to true if u is a string variable; otherwise it simplifies to false. For example, "a=b" and "Derive" are string variables.

SUM?(u) simplifies to true if u is a sum; otherwise it simplifies to false. For example, x+y and x-y are sums.

PRODUCT?(u) simplifies to true if u is a product; otherwise it simplifies to false. For example, x·y and x/y are products.

One of Marcelo's next desperate mails:

It seems that Microsoft does no longer accept programs with the old help facilities. I could solve it on Windows 7 by means of Winhlp32 but this strategy does not work any longer on Windows 11.

I could install Derive 6.10 but the help does not work. George sent me copied of the Derive files he took from Derive on Windows 7. It seems that Derive will not fully operate on Windows 11.

Regards,
Marcelo

But finally, I received the ultimate answer from our DERIVE-WIN-expert Günter Schödl:

Dear Josef,

I have now received a new Win11 computer (Lenovo) for testing. I have on it

1. Derive 6.1 installed with right click as administrator
2. the winhlp32 with the install routine (file install) I have already provided for win10, also installed as administrator after right click.

Conclusion: Everything works, algebra, graphics 2D and 3D and the help file.

If you do not try to install as administrator, you will fail because of the write permissions in the Windows directory.

Regardless of your rights (it does not matter whether you are logged in as administrator), you must select "Run as administrator" after right-clicking on the file!

Happy Marcelo wrote:

Dear Josef,

Thank you very much, it works fine now. I had failed to install Winhlp32 properly. Even Derive 5 works nicely.

Merry Christmas and happy new year.

Marcelo

Customized (DOS)DERIVE Menu

Ross Barnes wrote:

Dear Josef,

In DNL #16 at the bottom of page 3 there was a query from Glyn D Williams about customizing the menu of Derive for DOS 3 with file "DERIVE.MEN". You replied on the top of page 4:

"DNL:I have sent Mr Williams the information he wants and I announce for the next DNL to publish the way to "tune" one's own menu."

I've reviewed DNL #17 - DNL #25 but have been unable to find the information about customizing the menu of Derive for DOS. I also reviewed "dnl_index121.pdf" with no results.

Do you know where I can find information on menu customization for Derive for DOS 3/4?

One more question: Is there a comprehensive bug list for Derive 6.1? Reading through your DNLs and the JISC MAIL Derive-News archive I've seen mention of various minor problems with 6.1 but I've never seen a comprehensive list.

I use Derive for DOS 4.11 on my PC DOS computer as I slowly work my way through Numerical Analysis Via Derive by Schonefeld.

With your generosity I've been using Derive 6.1 as I read through Bernhard Kutzler's "Introduction to Derive 6" and various DNL articles.

Thank you for all your work on the Derive Users Group and the newsletters.

Happy holidays and best wishes for the new year,

Ross.

Dear Ross,

DNL#16 was very long ago – and you are right, I promised explaining how to customize the DERIVE menu. But unfortunately enough, I did not keep my promise.

When I read your mail I didn't remember DNL16 (from 1994 which is nearly 30 years ago!!) but as I am a collector of books, papers – and files I undertook a research among my many DERIVE versions and files.

And finally, I was lucky I found a version containing a DERIVE.MEN and a DERIVE.LAN file which both do not appear in all other versions.

I had to run a DOS-Box to make DERIVE 3.14 alive – and really, I had a GERMAN Menu – that was it.

I attach three files: DERIVE.MEN, DERIVE.LAN and DERIVE.HLP. I don't know if they work with DERIVE 4.11, too.

Try to include the *.MEN and the *.LAN in your DERIVE folder and then run DERIVE.

I believe that these files are not included in your DERIVE 4.11. If so, then rename them that you will not lose the original files. The HLP-file might be included, so please rename it e.g. as DERIVE.hpl.

The you will find a German version of the menu and parts of the other two files. You can load them into your editor (DOS-editor) and study its structure – and you can change as you like. I didn't change the structure but only translated into German.

So, much luck and I hope that this might help.

How far have you gone with Schonefeld's book so far? If I can provide any support, then feel free to ask.

Best regards and many thanks for bringing back my memories,

Ross answered:

Dear Josef,

thank you for the files! Your files loaded without any problems on my DfD 4.11 software.

I have attached an English version of DERIVE.MEN (DRVENGL.MEN) identical (hopefully) to the original DfD 4.11 menus. Perhaps one day another English speaking Derive user will send you a query about the menus.

Without your German DERIVE.MEN it would have been IMPOSSIBLE for me to build this file. I haven't made any changes to the original menu ordering or text. I'll need more user experience with DfD before I do that.

So far I'm not having any trouble the Schonefeld book. I took a Numerical Methods course forty years ago and read occasional articles since then mostly regarding old HP Calculators. I want to use the book to get me in a "Derive" functional programming state of mind as well as refresh my memory of various numerical methods.

Thanks again for all your assistance.

Sincerely,

Ross

P.S. I almost forgot...did anyone every compile a bug list for Derive 6.10?

Who Plays Video Games?

Tania Koller, St. Pölten, Austria^[1]

To be successful in business, collection of information is indispensable. Statistics is concerned with how best to collect, analyze, and interpret data. The following tasks serve to deepen the knowledge from descriptive and inferential statistics. Knowledge of the most important probability distributions is required.

137 students between their ages 15 and 17 years were surveyed about their behavior toward video games. Since the abundance of data would otherwise have been difficult to manage, the TI-92 was used for analysis.

This took into account the fact that in practice statistical analyses are now in almost all cases carried out by computer. Commercial statistical program packages are available for performing the calculations and creating graphs. However, many of these tasks can also be handled well with the support of the TI-92.

The simpler statistical procedures are always formulaic, so you could also do the evaluation by hand. However, in case of large amount of data in practice, no one would think of actually performing such calculations manually. The students know that, too. They find data analysis without technical support time-consuming, cumbersome and therefore boring, while they can be motivated to work with an appropriate calculator.

Some tasks, such as simulating random sampling using random numbers, would not have been possible at all without TI-92 or an appropriate computer program.

The Questionnaire:

1. How much time did you spend playing video or computer games in the last week?

No time hours

2. Do you like to play video and/or computer games?

Very much Reasonably Not really Not at all

I have never played → goto Question 9

3. What types of games do you play? (Mark all that apply.)

Action (Doom, Street Fighter, ...)

Adventure (King's Quest, Myst, Return to Zork, Ultima, ...)

Simulation (Flight Simulator, Rebel Assault, ...)

Sport (NBA Jam, Ken Griffey's MLB, NHL '94, ...)

Strategy, Puzzle, ...

^[1] With Tania's friendly permission I translated her German report and added the treatment of her data with TI-NspireCAS. Many thanks to my many years' colleague on the HAK St. Pölten, Lower Austria. All possible errors are in my responsibility

4. Why did you mark the games above? (Choose at most three of them.)
- I like the graphics / the realism.
 - Relaxation / recreation / entertainment
 - It improves my hand eye coordination.
 - It is a challenge for my mind.
 - It is a great feeling to win a game.
 - I would play anything when I am bored.
 - Other reasons (Please indicate!):.....
5. Where do you usually play video/computer games?
- Arcade
 - At home On a system/console (Sega, Nintendo,...)
 - On a computer (IBM, MAC, ...)
6. How often do you play?
- Daily Weekly Monthly Semiannually Even rarer
7. Do you still find time to play when you have a lot to do (e.g. if there are many examinations)?
- Yes (I can't let it go) No (school first)
8. Do you think that games are important educationally?
- Yes, or all years spending with video games were wasted.
 - No, it had been better reading books.
9. What do you not like about video games? (at most three answers!)
- It takes too much time.
 - It is frustrating.
 - It is lonely.
 - There are too many rules to learn.
 - It is too expensive.
 - It is boring.
 - My friends don't play.
 - It is senseless.
 - Other reasons (Please indicate)
10. Your Gender: Male Female
11. Your age:

12. Was there a computer at home when you attended lower secondary?

Yes No

13. What do you think of mathematics?

I hate mathematics I like mathematics.

14. How many hours a week do you work for money?

15. Do you own a PC? Yes No

Is there a CD-Rom-drive? Yes No

16. Do you have an email account? Yes No

Statistical analyses can be performed on the basis of a wide variety of variables. A listing of some of the dates of the first 19 students is given to the right.

This data sheet is stored as **video** on the TI-92 Plus (Figure 1).

On the one hand, there are the **quantitative variables** with continuous measurements, such as the amount of time spent playing video games in the last week (question 1), the age of live (question 11), the working hours (question 14). In principle, these data can be measured with any precision and the current variable value can be assigned to them

Nr.	Zeit	Gerne	Arten von Spielen					Warum?		
			Action	Adve	Simu	Sport	Strateg	Grafik	Entspan	Koord
1	12	2	0	0	0	1	0	0	1	0
2	4	2	0	0	0	0	1	0	1	0
3	7	1	0	1	0	0	1	0	1	0
4	3	2	0	1	0	0	1	0	1	0
5	0	3	0	0	1	0	1	1	0	0
6	0	3	1	0	0	0	1	1	1	1
7	0	3	1	0	0	0	1	1	1	0
8	2	3	0	1	0	0	1	1	1	0
9	3	2	0	0	0	0	1	0	1	1
10	0	3	0	0	0	0	1	0	1	1
11	0	3	0	0	0	0	1	0	0	0
12	1	3	0	0	0	1	1	0	1	0
13	0	2	1	1	0	0	0	1	0	0
14	1	2	0	1	0	0	1	0	0	0
15	1	2	0	0	0	0	1	0	0	0
16	1	3	1	1	0	0	0	1	0	0
17	3	2	1	0	0	0	0	0	0	0
18	10	1	0	0	0	1	0	0	0	0
19	0	3	0	0	0	0	1	0	1	0

Figure 0

At the other hand there are **qualitative variables**, e.g. the gender (question 10), or the Classification in the types of games (question 3) or the frequency of playing (question 6). The assignment of numbers to such "**categorical**" variables is necessary at the latest when statistical analysis is to be performed using a computer.

DATA	Ze...	wi...	Ac...	Ab...	Si...	Sp...	St...	Gr...	Er...	Ha...	Stat
	c1	c2	c3	c4	c5	c6	c7	c8	c9	c10	
1	12	2	0	0	0	1	1	0	1	0	
2	4	2	0	0	0	0	1	0	1	0	
3	7	1	0	1	0	0	1	0	1	0	
4	3	2	0	1	0	0	1	0	1	0	
5	0	3	0	0	1	0	1	1	0	0	
6	0	3	1	0	0	0	1	1	1	1	
7	0	3	1	0	0	0	1	1	1	0	

Figure 1

All questions that required a yes/no response were assigned "1" for yes and "0" for no. Such an assignment is arbitrary and has no empirical meaning whatsoever. In this case, we speak of **nominal scaled variables**. In the special case here of only two categories, we speak of a **dichotomous variable**. Nominal scaled variables are very limited in their evaluation possibilities. Strictly speaking, they can only be subjected to a frequency count. The calculation of a mean value, for example, is meaningless.

Tania did her investigation in 2001. She agreed that I could reproduce her project with TI-Nspire of nowadays. The first problem appeared when I entered the data in a *List & Spreadsheet* App because for entering all data it would need 36 columns and the TI-Nspire allows only 26 (from A – Z). That’s not so bad, because there are some questions in the questionnaire with answers not treated statistically. So, I selected only questions 1, 2, 3, 4, 6, 8, 10 and 11 together with their respective answers.

	A time	B att...	C ac...	D ad...	E si...	F sp...	G str...	H gr...	I re...	J co...	K
=											
1	12	2	0	0	0	1	1	0	1	0	
2	4	2	0	0	0	0	1	0	1	0	
3	7	1	0	1	0	0	1	0	1	0	
4	3	2	0	1	0	0	1	0	1	0	
5	0	3	0	0	1	0	1	1	0	0	
6	0	3	1	0	0	0	1	1	1	1	
7	0	3	1	0	0	0	1	1	1	0	
8	2	3	0	1	0	0	1	1	1	0	
9	3	2	0	0	0	0	1	0	1	1	
10	0	3	0	0	0	0	1	0	1	1	
11	0	3	0	0	0	0	1	0	0	0	
12	1	3	0	0	0	1	1	0	1	0	
13	0	2	1	1	0	0	0	1	0	0	
14	1	2	0	1	0	0	1	0	0	0	
15	1	2	0	0	0	0	1	0	0	0	

I label this figure as *N-Figure 1* which corresponds with Figure 1 (TI-92).

N-Figure 1

The answers to question 2 (Do you like playing video games?) can be assigned numbers that reflect an order relation by clever coding:

Very much = 1 Reasonably = 2 Not really = 3 Not at all = 4 I have never played = 5

Such variables, where the code numbers used have an empirical meaning with respect to their order, are called **ordinally scaled** (ordinal variables).

This includes the variable of question 6.

Daily = 1 Weekly = 2 Monthly = 3 Semiannually = 4 Even rarer = 5

Now let's start with to the first questions:

- (1) Represent the age distribution of the students surveyed in a histogram.

The histogram can be defined by [F2] Plot Setup > [F1] Define.

The students' age can be found in column **c29**.

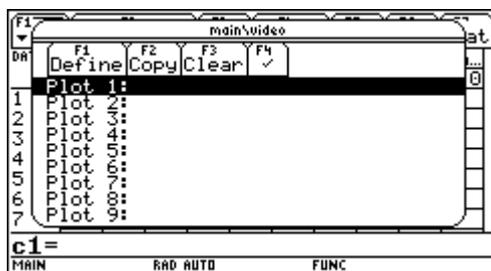


Figure 2

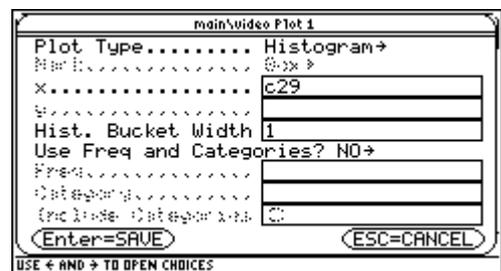


Figure 3

With [F2] 9 ZoomData a plotting area is suggested in the x-direction, but not in the y-direction. Since the best drawing area does not always result here, it is more favorable to set the area with [WINDOW] not only in the y-direction, but also in the x-direction by yourself.

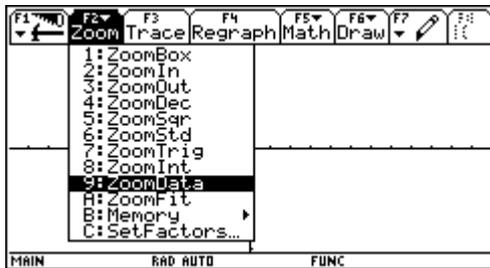


Figure 4

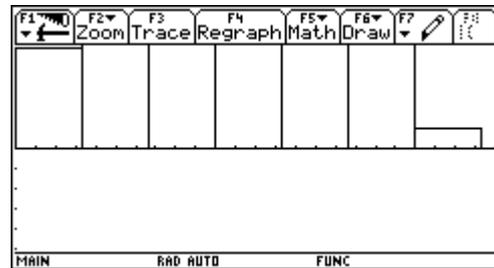


Figure 5

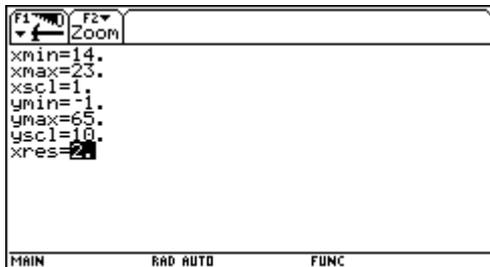


Figure 6

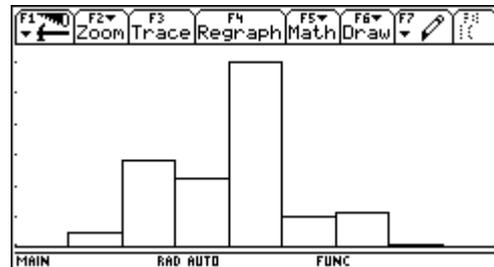


Figure 7

[F3]Trace and \odot can be used to display the absolute frequencies of the individual age groups. You can see, for example, that the oldest student surveyed was 21 years old.

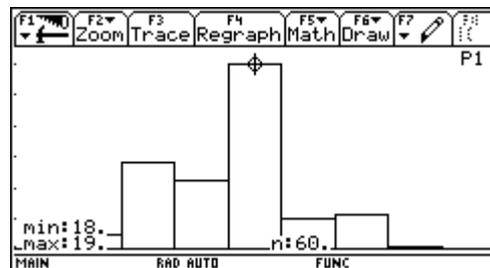


Figure 8

When setting the window coordinates, it should be noted that the **xmin** value has a special significance in this context. It indicates where the individual columns of the histogram begin. For example, in the following figure, **xmin** has been changed from 14 to 13.5.

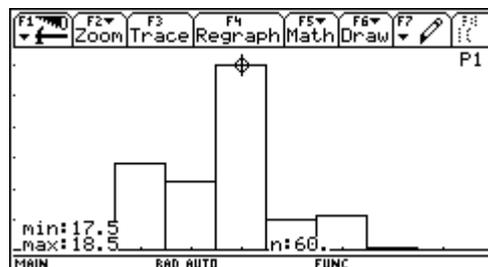
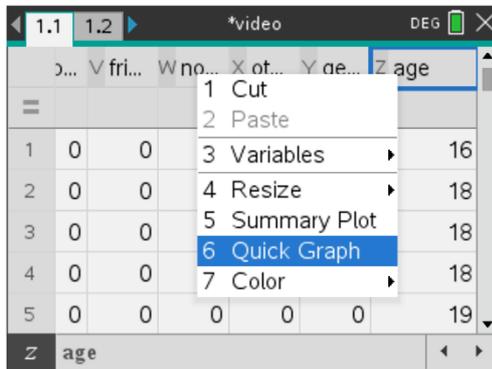


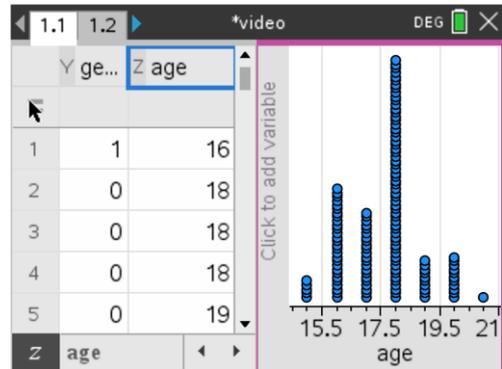
Figure 9

I proceed with the TI-Nspire using my spreadsheet. The columns don't have numbers but labels. **c29** corresponds with column **age**. *Quick Graph* shows immediately a bar chart providing appropriate boundaries in all directions.

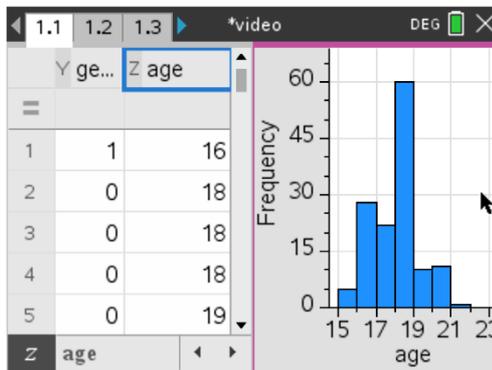
But you can also mark column age and then insert a *Data & Statistics App* to obtain a bar chart first which can be changed to a histogram. Width and Alignment can be adapted similar to the TI-92 procedure.



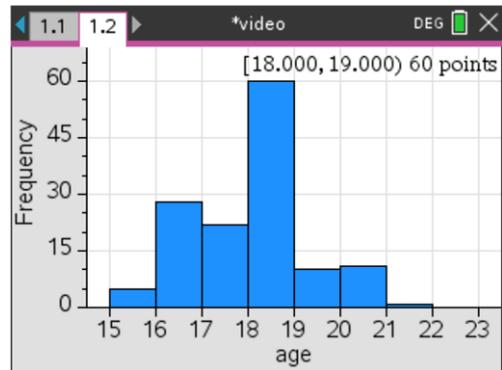
N-Figure 4



N-Figure 5



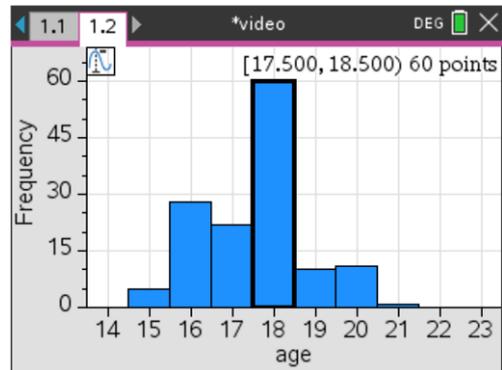
N-Figure 5a



N-Figure 7



N-Figure 8



N-Figure 9

(2) Plot the absolute frequencies of last week's game durations in a histogram, and interpret the result. The game durations are given in column **c1**.

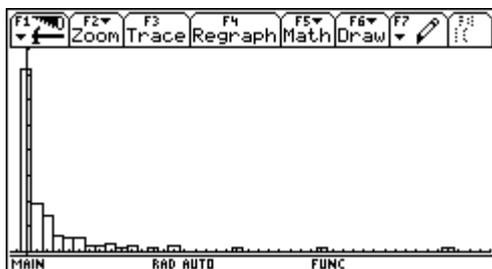
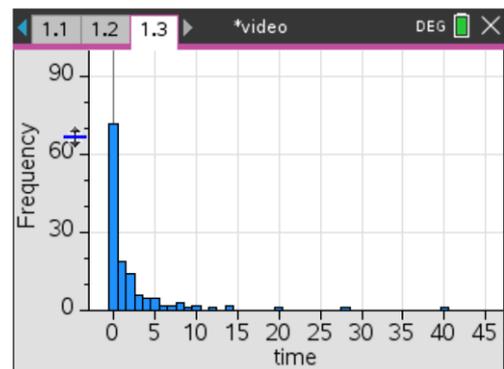


Figure 10



N-Figure 10

72 students of have not played at all in the last week, but there are also extreme outliers, namely one student each with 20, with 28 and with 40 hours of play.

- (3) Find the **mean** and **median**.

Explain the difference of these values in the concrete example.

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

The mean of n values x_i is the sum of all values divided by its number:

The median **medStat** is the value above and below which half of the ordered measured values lie.

These values are obtained from the data sheet via **F5** Calc > Calculation Type > OneVar. You can also recall these values later under **F7** Stat.

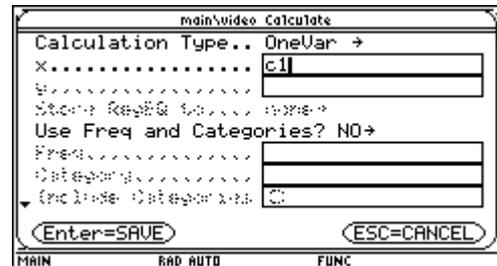


Figure 11

Thus, students played an (arithmetic) mean of 2.3 hours in the last week, but the median is 0 hours.

This is not surprising, as we already know that out of 137 students surveyed, 72 had not played at all in the past week.



Figure 12

The large value of the arithmetic mean is caused by the few students playing extremely long. When outliers occur, the calculation of the arithmetic mean is usually not useful.

Inspecting the data, one finds that the student in row 126, with 40 hours of play time, is the most extreme "outlier" in this data series.

In order to examine the effect this value, we copy column **c1** into the still free column **c36**. This is done with **F4** Header > **c36=c1** **ENTER**.

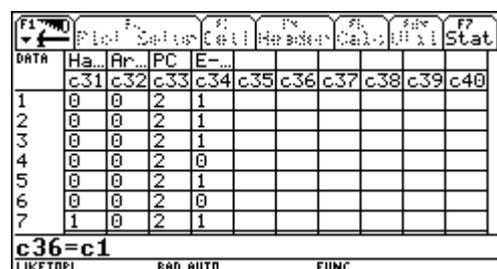


Figure 13

Afterwards, **c1** is removed by typing **←** **ENTER** as well, so that the data in **c36** can be changed.

(I do this because I'd like to avoid losing data when experimenting with the values. You can change the value of **r126c1**, too, and then after performing OneVar Stats again undo the change.)

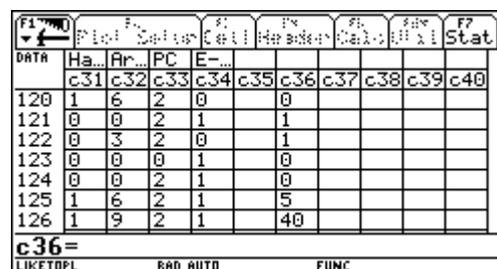


Figure 14

Let's assume that student #126 did not play one single hour last week, we must replace the corresponding value in **r126c1** by 0.

How does this change affect mean and median?

The mean is now 2.0 hours, so it has immediately decreased by 18 minutes.

The median remains unchanged, of course, since it cannot become less than zero. The median is completely insensitive to outlier values. It is also completely irrelevant how large the largest measured value is.

DATA	Ha...	Ar...	PC	E...	F...	F...	F...	F...	F...
	c31	c32	c33	c34	c35	c36	c37	c38	c39 c40
121	0	0	2	1	1				
122	0	3	2	0	1				
123	0	0	0	1	0				
124	0	0	2	1	0				
125	1	6	2	1	5				
126	1	9	2	1	0				
127	1	0	2	1	0				

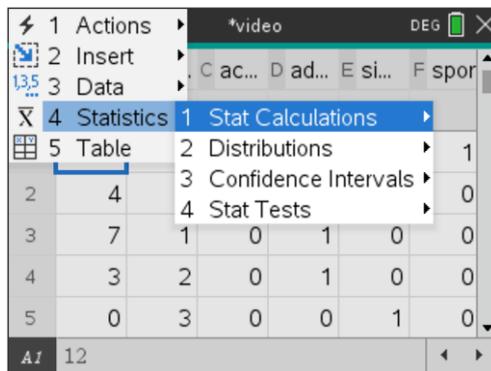
Figure 15

DATA	Ha...	Ar...	PC	E...	F...	F...	F...	F...	F...
	c31	c32	c33	c34	c35	c36	c37	c38	c39 c40
121	0								
122	0								
123	0								
124	0								
125	1								
126	1								
127	1								

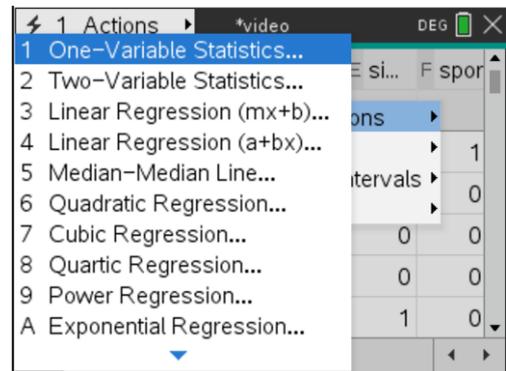
STAT VARS	
\bar{x}	=2.00365
Σx	=274.5
Σx^2	=2691.25
Sx	=3.96793
nStat	=137.
minX	=0.
q1	=0.
medStat	=0.

Figure 16

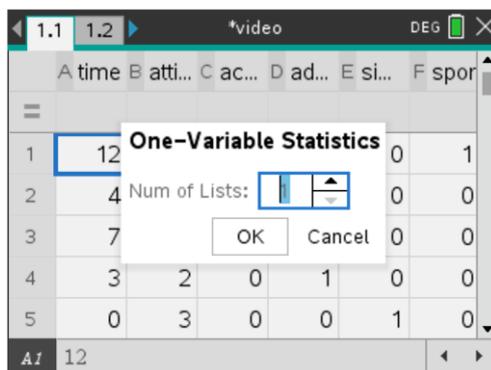
Remark. As we don't have a free column to copy c1 as shown on the TI-92 we must overwrite the 2nd and the 3rd column for presenting the statistics result.



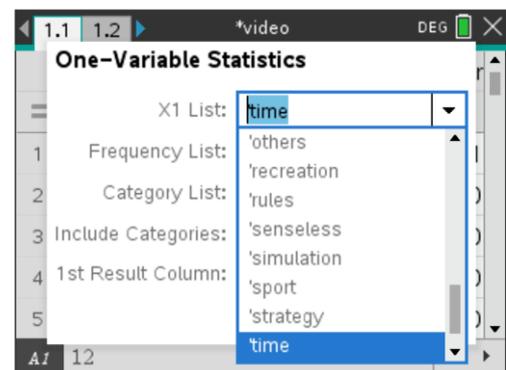
N-Figure 11



N-Figure 11a



N-Figure 11b



N-Figure 11c

Next page shows the One-Variable Statistics on the TI-Nspire computer screen for both cases.

A	time	B	C	D	ad...
=			=OneVar('time,1): CopyV		
1	12	Title	One-Variable Statistics	0	
2	4	\bar{x}	2.29562	0	
3	7	Σx	314.5	1	
4	3	Σx^2	4291.25	1	
5	0	$s_x := s_{n-1}x$	5.12296	0	
6	0	$\sigma_x := \sigma_{n}x$	5.10423	0	
7	0	n	137.	0	
8	2	MinX	0.	1	
9	3	Q_1X	0.	0	
10	0	MedianX	0.	0	
11	0	Q_3X	2.	0	
12	1	MaxX	40.	0	
13	0	$SSX := \Sigma(x-\bar{x})^2$	3569.28	1	

N-Figure 12

A	time	B	C	D	ad...
=			=OneVar('time,1): CopyV		
1	12	Title	One-Variable Statistics	0	
2	4	\bar{x}	2.00365	0	
3	7	Σx	274.5	1	
4	3	Σx^2	2691.25	1	
5	0	$s_x := s_{n-1}x$	3.96793	0	
6	0	$\sigma_x := \sigma_{n}x$	3.95342	0	
7	0	n	137.	0	
8	2	MinX	0.	1	
9	3	Q_1X	0.	0	
10	0	MedianX	0.	0	
11	0	Q_3X	2.	0	
12	1	MaxX	28.	0	
13	0	$SSX := \Sigma(x-\bar{x})^2$	2141.25	1	

N-Figure 16

- (4) Now it is of course also interesting how the gender of the students influences their playing time per week. For this purpose, the box diagrams for all students together and separately according to gender are to be shown.

In addition to the median, which divides an ordered list of measured values in such a way that 50% of the values lie above and 50% below the median (also called the second quartile), there are two other distinguished points: the first (lower) quartile and the third (upper) quartile. Below the first quartile and above the third quartile, 25 % of the ordered measurements are located in each case.

The lower and upper quartiles as well as the median are entered in the **box-plot diagram**. The 1st, 2nd and 3rd quartiles divide the measurements into four parts with equal frequencies. The distance between 1st and 3rd quartile (box) covers 50% of the values and is a measure for the dispersion of the values. To the left and right of the box, one also applies so-called whiskers (whiskers of a cat).

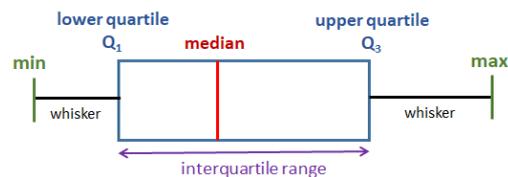


Figure 17

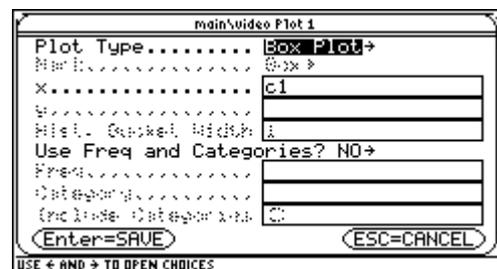


Figure 18

Box-plot diagrams show in which areas the individual values occur and with which frequencies. The box-plot diagram can be defined again starting from the data sheet. But it is also possible from the [Y=]-Editor

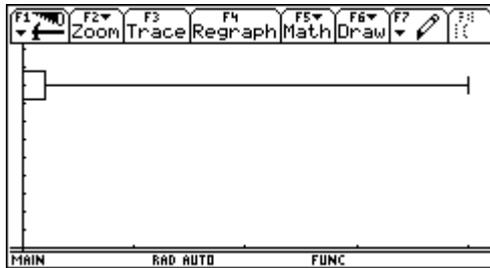


Figure 19

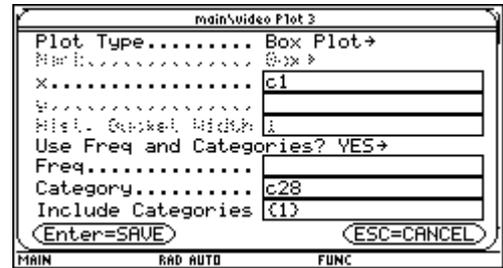


Figure 20

For the other two diagrams, a distinction must now be made according to gender. The gender is in column **c28**, namely "female" was assigned the number "0" and "male" was assigned the number "1". Therefore, when defining the new box diagram, "Yes" is entered for the question **Use Freq and Categories?** for Category enter **c28**, for **Include Categories** {0} and {1}, respectively.

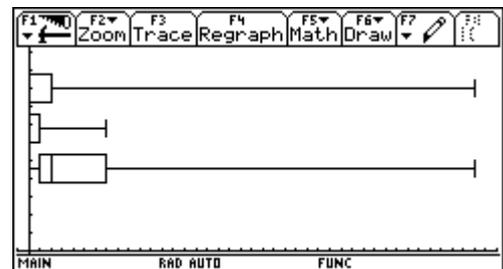


Figure 21

In order from top to bottom, the playing times of all students, female students and male students are given here.

With **F3** Trace you can query the individual quartile values in sequence. The first two figures look atypical, because here the minimum value **minX** coincides with the first quartile **q1** and the median **med**.

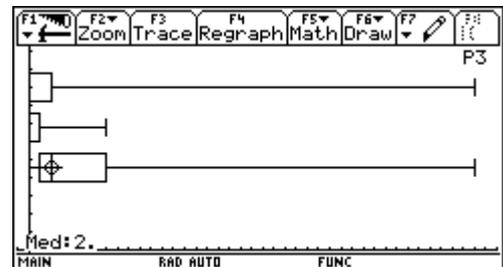
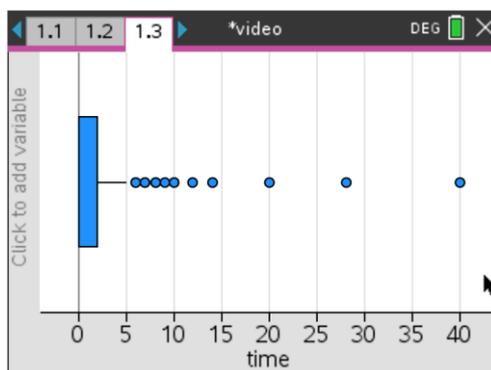
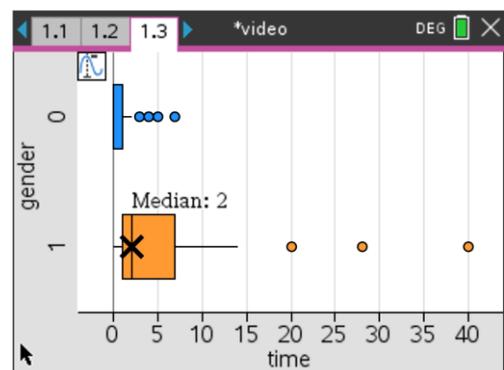


Figure 22

The third quartile for female students is the same size, one hour, as the first quartile for male students. The female students played a maximum of 7 hours in the last week, which corresponds to the 3rd quartile of the male students, i.e. a quarter of the male students played longer.



N-Figure 19



N-Figure 22

- (5) Represent the answers to (2). (Do you like playing video and/or computer games?) as a histogram. Since this is an ordinal scaled variable, the responses are each assigned a specific code number that corresponds to the order of the responses.

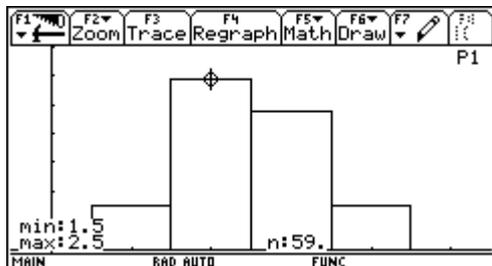
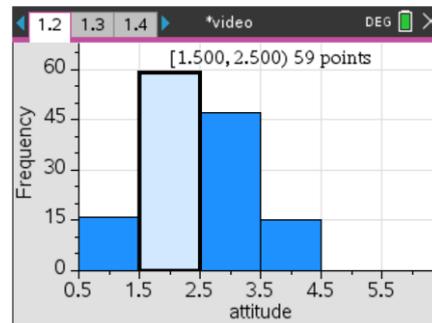


Figure 23



N-Figure 23

The histogram is excellently suited for determining the frequencies of the individual answers, since the absolute frequencies can be read directly with $\boxed{F3}$ Trace. It is interesting that all students state that they have played video games at any time.

Answers	Code	absolute frequency	relative frequency
Very much	1	15	11 %
Reasonably	2	59	43 %
Not really	3	48	35 %
Not at all	4	15	11 %
Never played	5	0	0 %

- (6) Which of the questions has an ordinal variable, too?

In question 6 (How often do you play?) you can also scale the answers according to their order.

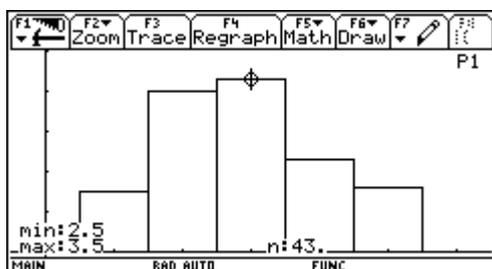
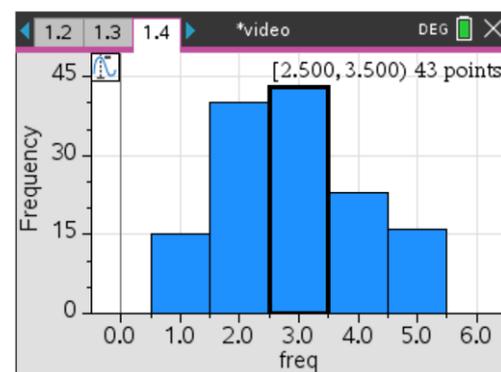


Figure 24



N-Figure 24

Answers	Code	absolute frequency	relative frequency
Daily	1	15	11 %
Weekly	2	40	29 %
Monthly	3	43	31 %
Semiannually	4	23	17 %
Even rarer	5	16	12 %

Of course, it is useless to calculate the arithmetic mean in this example, but it is possible to obtain information from the median even for ordinal scaled variables under certain circumstances



Figure 25

- (7) Inspecting the results of (5) and (6), the question arises as to the correlation of the variables. Are the 11% of students who like to play the same 15 ones who play daily? Are the 11% of students who don't like video games at all in the group of those who play less than 1 time per semester?

It is therefore a matter of analyzing the relationship between two variables of the type "the larger the one variable, the larger the other". This only works if the variables are at least ordinal scaled or dichotomous.

It is recommended to first create a scatter plot. In our example, the frequency of the games is plotted against the popularity of the games:

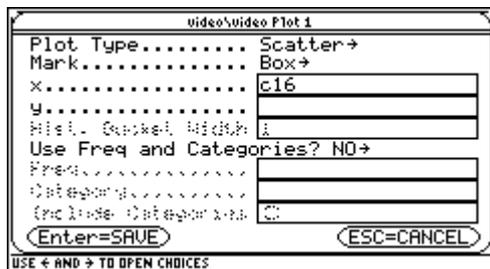


Figure 26

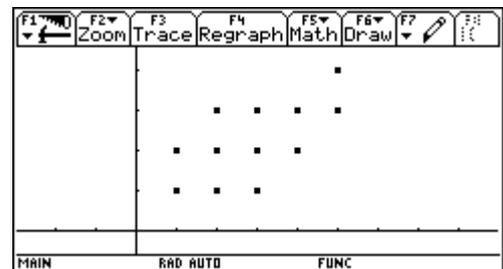


Figure 27

The diagram gives the impression of a linear relationship.

The strength of the correlation can be described by a measure called the **correlation coefficient r** (on the TI-92 **corr**). The strength of the correlation is determined by the magnitude of the correlation coefficient. The sign indicates whether the correlation is equal or opposite. The following classification is common

Korrelationskoeffizient	Classification
$ r \leq 0.2$	very weak correlation
$0.2 \leq r \leq 0.5$	weak correlation
$0.5 \leq r \leq 0.7$	moderate correlation
$0.7 \leq r \leq 0.9$	strong correlation
$0.9 \leq r \leq 1$	very strong correlation

So, we perform a **linear regression** on the TI-92:

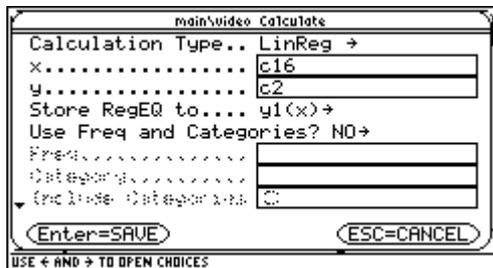


Figure 28



Figure 29

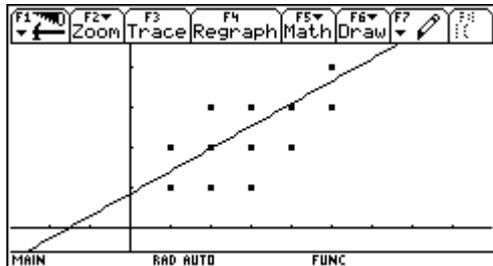
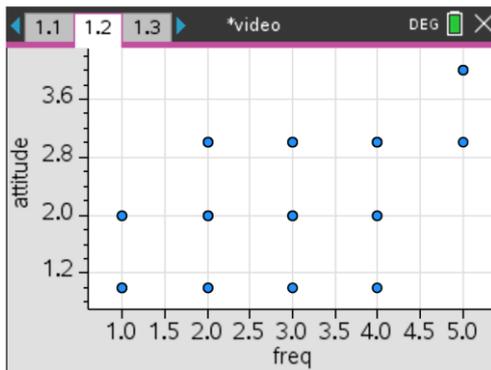


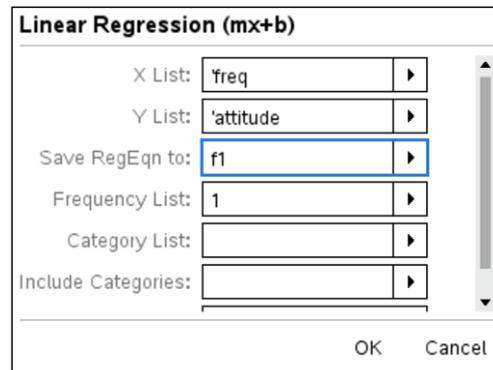
Figure 30

The correlation coefficient is $r = 0.787$, which means that there is a high - positive - correlation.

Here you can see the scatter diagram on the TI-Nspire handheld screen followed by the calculation of the linear regression:



N-Figure 27

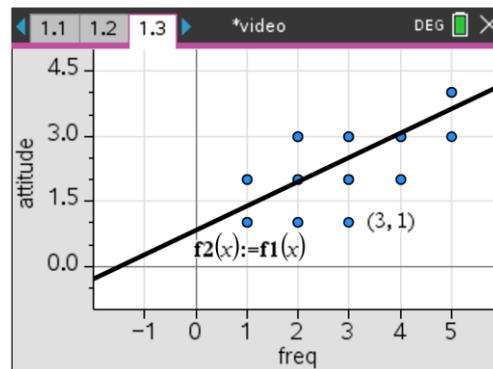


N-Figure 28

I copied the respective columns to second *Lists & Spreadsheet App* within the same problem in order to not overwrite columns in my original spreadsheet. The equation of the regression line is stored as **f1(x)** which then can be plotted in addition to the scatter diagram.

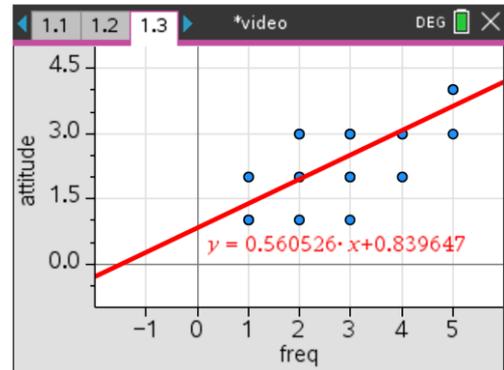
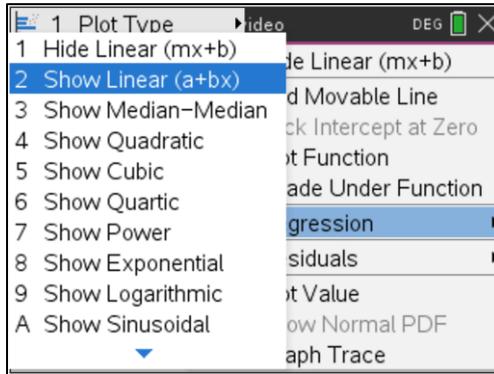
	A	atti...	B	freq	C	D	E
=						=LinRegV	
2		2	2	RegE...	m*x+b		
3		1	1	m	0.560526		
4		2	2	b	0.839647		
5		3	3	r ²	0.619355		
6		3	4	r	0.786991		
D1	="Linear Regression (mx+b)"						

N-Figure 29



N-Figure 30

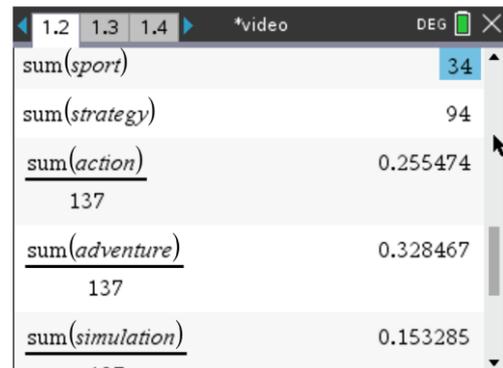
If you are not interested in the correlation but only want to receive the regression line you can immediately get it via Menu > Analyze > Regression > Show Linear $mx + b$.



- (8) With nominally scaled variables, a number is assigned to the categories to enable the use of the computer, but the assignment is purely arbitrary. Therefore, nominally scaled variables are severely limited in their ability to be analyzed. Actually, they can only be used for frequency counting. This includes, for example, the question about the types of games that are played (question 2). There were five types given and all applicable ones were to be marked, with each cross being assigned "1" and each missing cross "0".



Figure 31



N-Figure 31

Action	Adventure	Simulation	Sport	Strategy
25,5 %	32,8 %	15,3 %	24,8 %	68,6 %

Now, it is also interesting to know whether this question was answered by all respondents, or whether there are also students who ticked more than one type of game. Since the answers were stored in the columns **c3**, **c4**, **c5**, **c6** and **c7**, only the numbers of these columns have to be added. If the sum is 0, no game was ticked, if it is 1, one game was ticked, and so on.

Therefore, the sum of **c3**, **c4**, **c5**, **c6** and **c7** is calculated in column **c36**. To be able to order the values of the column also according to their size, the column **c36** is also entered into the column **c37**.

	F1	F2	F3	F4	F5	F6	F7
	Plot	Setup	Cell	Header	Calc	Util	Stat
DATA	E-mail						
	c34	c35	c36	c37	c38		
1	1		2				
2	1		1				
3	1		2				
4	0		2				
5	1		2				
6	0		2				
7	1		2				
c36=c3+c4+c5+c6+c7							
MAIN RAD AUTO FUNC							

Figure 32

	F1	F2	F3	F4	F5	F6	F7
	Plot	Setup	Cell	Header	Calc	Util	Stat
DATA	E-mail						
	c34	c35	c36	c37	c38		
1	1		2				
2	1		1	1			
3	1		2	2			
4	0		2	2			
5	1		2	2			
6	0		2	2			
7	1		2	2			
c37=c36							
MAIN RAD AUTO FUNC							

Figure 33

To change the column, you have to delete **c36** in the header, the content of the column remains unchanged. By selecting **[F6] Util 3: SortColumn** the content of the column is now sorted by size.

	F1	F2	F3	F4	F5	F6	F7
	Plot	Setup	Cell	Header	Calc	Util	Stat
DATA	E-mail						
	c34	c35	c36	c37	c38		
1	1		2	2			
2	1		1	1			
3	1		2	2			
4	0		2	2			
5	1		2	2			
6	0		2	2			
7	1		2	2			
c37=							
MAIN RAD AUTO FUNC							

Figure 34

	F1	F2	F3	F4	F5	F6	F7
	Plot	Setup	Cell	Header	Calc	Util	Stat
DATA	E-mail						
	c34	c35					
1	1		2	2			
2	1		1	1			
3	1		2	2			
4	0		2	2			
5	1		2	2			
6	0		2	2			
7	1		2	2			
c37=c36							
TYPE OR USE ↔+↔+ CENTER)=BK AND [ESC]=CANCEL							

Figure 35

With this, we can see (count) that 16 students did not mark any type, 46 marked one type, 48 marked two types, 23 marked three types, 2 marked four types, and 2 marked all five types of video games.

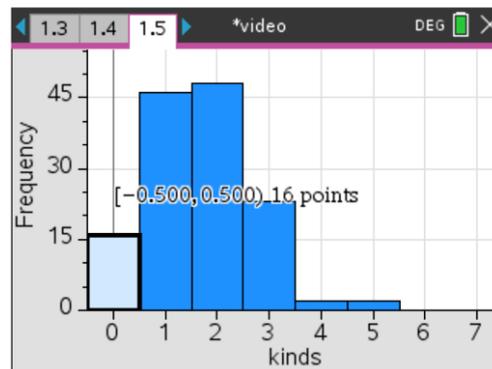
	F1	F2	F3	F4	F5	F6	F7
	Plot	Setup	Cell	Header	Calc	Util	Stat
DATA	E-mail						
	c34	c35	c36	c37	c38		
13	1		2	0			
14	1		2	0			
15	1		1	0			
16	0		2	0			
17	0		1	1			
18	1		1	1			
19	0		1	1			
r19c37=1							
MAIN RAD AUTO FUNC							

Figure 36

On the TI-Nspire I copy the columns of the game types into my second spreadsheet app (cols E – I) and add the values in the rows in column (kinds). After producing the histogram (or dot plot) I can trace the numbers of the students.

	E	F	G	H	I	J
1	0	0	0	1	1	2
2	0	0	0	0	1	1
3	0	1	0	0	1	2
4	0	1	0	0	1	2
5	0	0	1	0	1	2
J1 =e1+f1+g1+h1+i1						

N-Figure 34



N-Figure 36

It is not surprising that 16 students did not tick any type of game, since 16 students also stated that they play less than once per semester. Of course, you can't be sure if you haven't checked it. You could now look in the questionnaires or in the table to see if this is true. Or you could check how often the students play who did not specify a type:

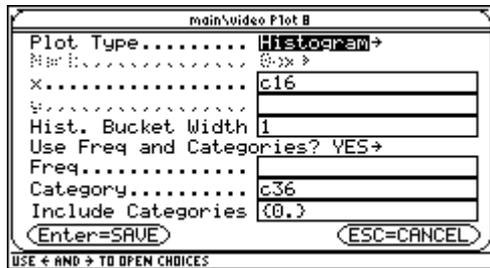


Figure 37

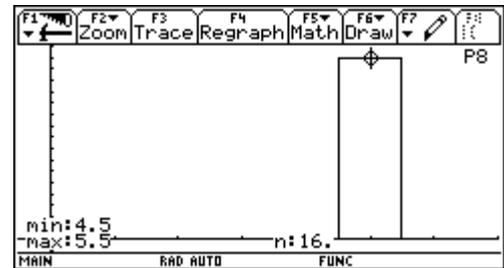
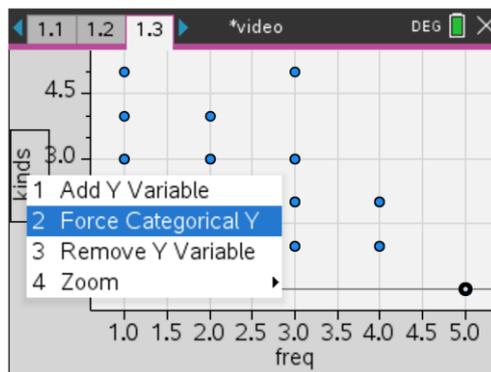
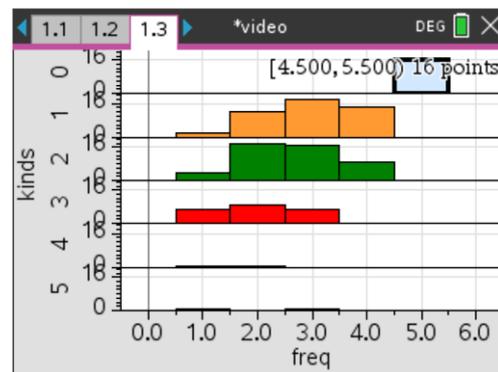


Figure 38

The plot of the categories on the TI-Nspire looks quite different, because it is not able to select only one category (0) – at least I was not able to do it. We find all categories collected in one graph and we have to select the requested one:



N-Figure 37



N-Figure 38

It turns out that our conjecture is indeed the reason.

(9) The question why video games are played was answered as follows:

I like the graphics / the realism.	34,3 %
Relaxation / recreation / entertainment	47,4 %
It improves my hand eye coordination.	21,9 %
It is a challenge for my mind.	38,7 %
It is a great feeling to win a game.	39,4 %
I would play anything when I am bored.	29,9 %
Other reasons	3,6 %

(10) What do you not like about video games? The answers were:

It takes too much time.	46,7 %
It is frustrating.	13,1 %
It is lonely.	30,7 %
There are too many rules to learn.	8,0 %
It is too expensive.	50,3%
It is boring.	19,7 %
My frnds don't play.	7,3 %
It is senseless.	19,0 %
Other reasons	4,4 %

(11) How many of the students are female?

48 of $N = 137$ students are male, i.e. 35%,
so $M = 89$ are female, which is 65%.



Figure 39

We will use our material to do some probability theory:

From $N = 137$ students, $n = 40$ are to be selected.

When selecting the 40 students, it is a "sampling without replacement". So, it is a hypergeometric distribution. We "expect" to be 26 female students in a sample, i.e. 65%. What is the probability that in a sample of $n = 40$ students exactly $k = 26$ (65 %) are female? Applying hypergeometric distribution, the probability is 15.6 %.

If the population is relatively large and the sample relatively small, the hypergeometric distribution may be approximated by the binomial distribution. Then the probability is 13.1 %.

As a consequence of the central limit theorem, the binomial distribution and the hypergeometric distribution differ very little from a normal distribution if the sample size n and, in the case of the hypergeometric distribution, the ratio N/n are large enough.

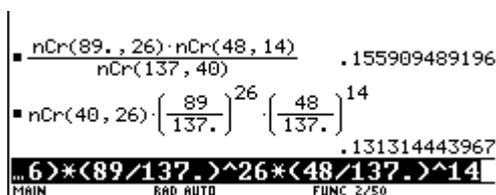
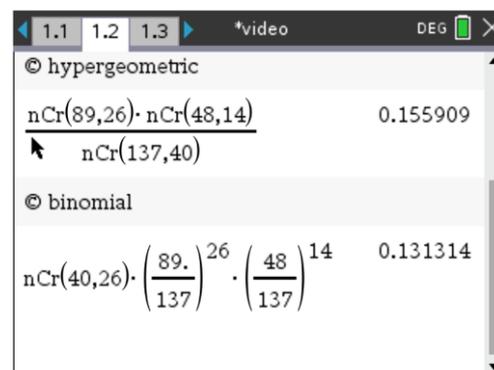


Figure 40



N-Figure 40

The following rule of thumb is often used: If the random variable X is

- binomial distributed with parameters n and p and if $\sigma^2 = n \cdot p \cdot (1 - p) \geq 9$
- or hypergeometrical distributed with parameters N , M , n and if

$$\sigma^2 = n \cdot \frac{M}{N} \cdot \left(1 - \frac{M}{N}\right) \cdot \frac{N-n}{N-1} \geq 9 \text{ and } N \geq 2n, \text{ then with } \mu = n \cdot p \text{ respectively } \mu = n \cdot \frac{M}{N}$$

$$\text{and } a < b \text{ applies: } P(a \leq X \leq b) = \Phi\left(\frac{b+0.5-\mu}{\sigma}\right) - \Phi\left(\frac{b-0.5-\mu}{\sigma}\right).$$

In our example is $\sigma^2 = n \cdot p \cdot (1-p) = 40 \cdot 0.65 \cdot 0.35 = 9.1 \geq 9$

but $\sigma^2 = n \cdot \frac{M}{N} \cdot \left(1 - \frac{M}{N}\right) \cdot \frac{N-n}{N-1} = 40 \cdot \frac{89}{137} \cdot \left(1 - \frac{89}{137}\right) \cdot \frac{137-40}{136} = 6.5 \leq 9$ and $137 > 80$

i.e. in our case the binomial distribution is not a good approximation for the hypergeometric distribution. Using the normal distribution, you get 13.16%, which is a good approximation for the binomial distribution. We accept the poor fit to the approximation for the hypergeometric distribution in the further treatment of this example.

We are looking for a **95% estimation interval** for the absolute or relative proportion of female students in this sample. This is the range in which the absolute or relative frequency of girls lies with a probability of 95%.

The approximation equation $P\left(|h_n - p| \leq \frac{z \cdot \sigma}{n}\right) \approx 2\Phi(z) - 1$ applies.

Now $2\Phi(z) - 1 = 0.95$, so $z = 1.96$.

$$\sigma^2 = n \cdot p \cdot (1-p) = 40 \cdot \frac{89}{137} \cdot \frac{48}{137} = 9.1$$

$$\frac{z \cdot \sigma}{n} = \frac{z}{n} \sqrt{n \cdot p \cdot (1-p)} = z \sqrt{\frac{p \cdot (1-p)}{n}} \approx 0.148$$

The 95% estimation interval is [0,502; 0,798] and [20,32] for the absolute frequencies.

- (12) The percentage of female students should be determined. We simulate samples of 40 people from our population in the data sheet video with a TI-92 program and examine the proportion of girls in them. For this purpose, we write a small program:

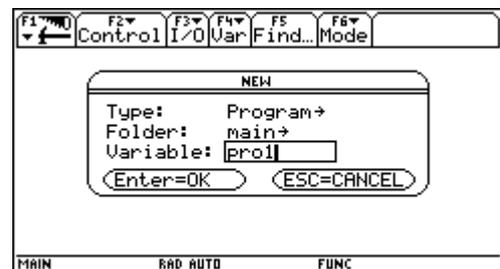


Figure 42

Via **APPS** 7:ProgrammEditor > 3:New we open a new program named **pro1**.

g is the size of the population; n is the size of the sample. With **seq(0,x,1,g)** we create a list of length g with all zeros and store it under the name **livid**. **rand(g)** generates a random number from 1 to g . This command is used to randomly select one person from the population.

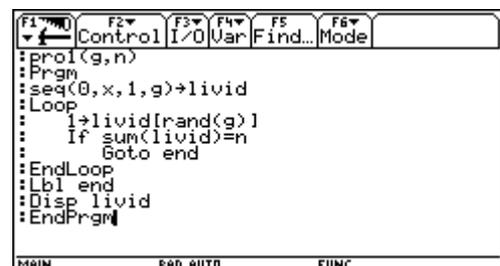


Figure 43

The value with the place number determined with **rand(g)** is assigned the number "1" in the list **livid**. This is done with **1→livid[rand(g)]**. This is repeated until the desired sample size is reached. **livid** then has exactly n ones and the rest is all zeros. The persons who receive "1" are selected.

For testing **pro1(8,5)** gives a list of 8 elements with 5 ones, **pro1(20,10)** gives a list of 20 elements with 10 ones, etc. **randseed number** initiates the random number generator. [F5] PrgmIO switches to the Program I/O screen and presents the latest results for the lists (named **livid** in **pro1**).

livid in the [HOME]-screen calls the last program output.

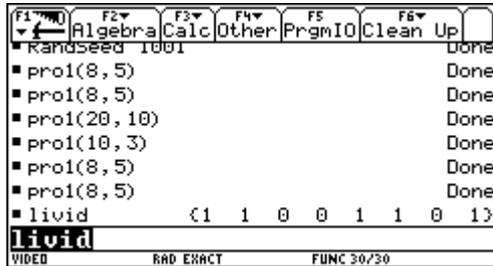


Figure 44

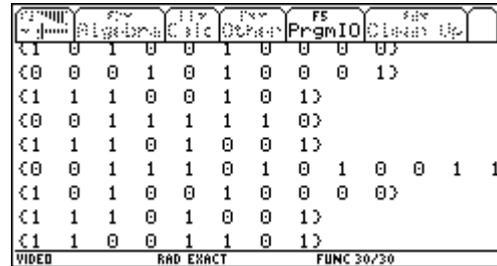


Figure 45

This list is stored in the table **video** in an empty column (here in **c35**).

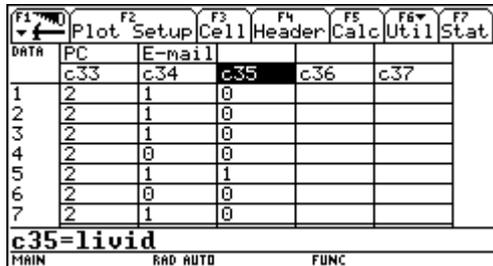


Figure 46

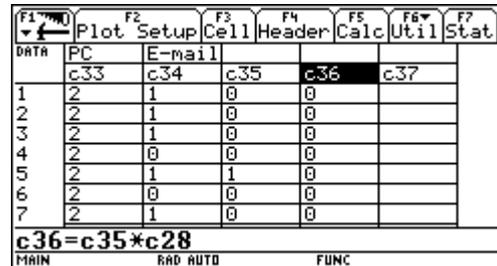


Figure 47

Column **c28** indicates the sex of the individuals in the population, and "1" stands for male and "0" for female.

Column **c36** is **c28 · c35**. So, only rows with male students are indicated by a "1". Then the sum of this column gives the number of the male students in the sample and all others are female ones.

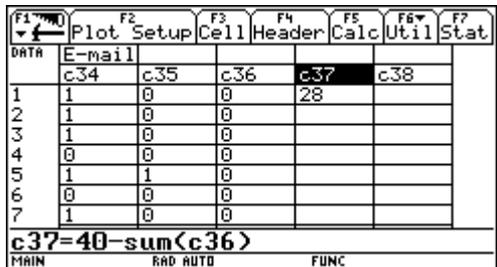


Figure 48

However, it is not necessary to call up the data sheet for this purpose; the program can also be extended accordingly.

Enter **pro2(137,40)** in the Home screen and find the result in the Program I/O screen.



Figure 49

I skip presenting **pro1** and its application on the TI-Nspire. **pro1** is included in **video.tns** and you can perform the procedure using the second spreadsheet. You can ask your students to perform 50 runs of **pro1** or **pro2** and collect the data in a table. Don't forget to use various **randseed** numbers.

RandSeed 1002	Done	"pro2" stored successfully
pro2(137,40)	{ 30,0.75 }	Define pro2 (g,n)=
pro2(137,40)	{ 27,0.675 }	Func
pro2(137,40)	{ 25,0.625 }	Local livid,anz
pro2(137,40)	{ 26,0.65 }	livid:=seq(0,x,1,g)
pro2(137,40)	{ 27,0.675 }	Loop
pro2(137,40)	{ 25,0.625 }	livid[randInt(1,g)]:=1
pro2(137,40)	{ 25,0.625 }	If sum(livid)=n:Exit
pro2(137,40)	{ 25,0.625 }	EndLoop
		©Disp livid
		anz:=n-sum(livid·gender)
		{ anz, $\frac{1 \cdot \text{anz}}{n}$ }
		EndFunc

N-Figure 49

The following table shows the absolute and relative frequencies for 50 samples.

abs. fr.	26	25	24	28	25	24	29	27	24	23
rel. fr.	0.650	0.625	0.600	0.700	0.625	0.6	0.725	0.675	0.600	0.575
abs. fr.	24	28	27	24	25	29	28	28	27	26
rel. fr.	0.600	0.700	0.675	0.600	0.625	0.725	0.700	0.700	0.675	0.650
abs. fr.	29	26	28	26	24	26	24	30	26	28
rel. fr.	0.725	0.650	0.700	0.65	0.600	0.650	0.600	0.750	0.650	0.700
abs. fr.	26	23	25	25	23	25	29	25	24	28
rel. fr.	0.650	0.575	0.625	0.625	0.575	0.625	0.725	0.625	0.600	0.700
abs. fr.	23	22	26	23	23	26	25	24	25	22
rel. fr.	0.575	0.550	0.650	0.575	0.575	0.650	0.625	0.600	0.625	0.550

The absolute frequencies of the girls range from 22 to 30, with a mean of 25.6.

The frequencies are displayed in a histogram and the corresponding normal distribution curve is plotted.



Figure 50

Decisive for the answer to the question whether the given frequency distribution of the values of a variable is considered as normally distributed is the fact whether this distribution differs significantly from a normal distribution or not (χ^2 -test, Kolmogorow-Smirnov-test). These tests will not be discussed here.

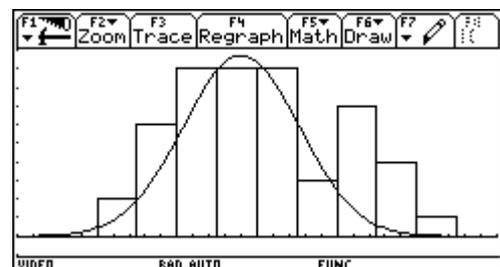
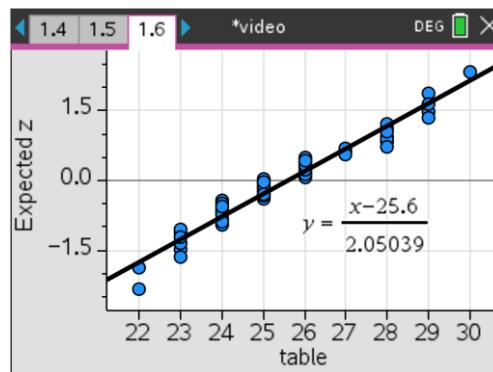
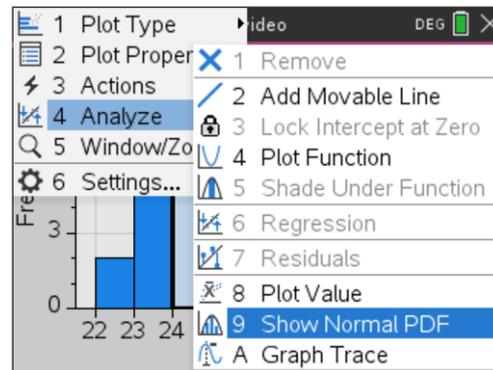
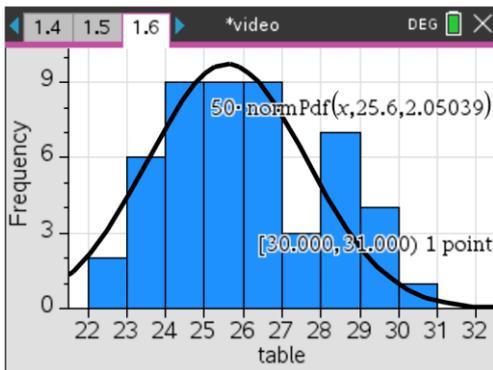


Figure 51



N-Figures 51

(13) Using a TI92 program **pro3(g,n)** - similar to **pro2(ng)** from (12) -, random samples with increasing sample size were drawn. The results are entered in the table.

Sample size	Relative freq. of girls in the sample
5	0,600
6	0,667
7	0,714
8	0,750
9	0,778
10	0,700
20	0,650
30	0,633
40	0,800
50	0,680
60	0,667
70	0,600
80	0,638
90	0,667
100	0,640
110	0,672
120	0,675
130	0,646

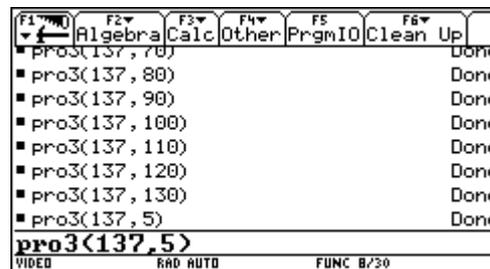


Figure 52

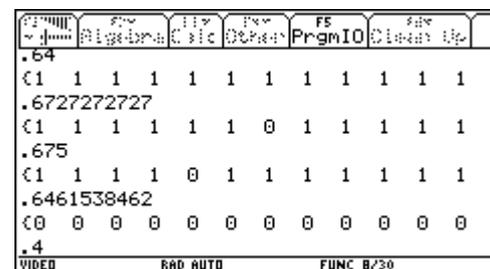


Figure 53

As known, 89 of the 137 students are female, i.e. $p = 0.650$.

Of course, the results in the table are purely random. if the experiment is repeated the table will look completely different.

As the sample size increases, the proportion of girls in the population (65%) is better and better approximated, but these approximations depend on chance. We now want to know how much we can trust a certain sample with a certain sample size and how large we should reasonably choose the sample size.

- (14) Let's assume that you know only 50 students who participated in this survey. Of these, 34 are female, or 68%. Thus, you would estimate that the percentage of girls in the total student respondents (all 137) is also 68%. We now want to find out how reliable such an estimate is. This is a typical problem where a sample is to be inferred to the entirety. One can specify an interval within which the actual girl percentage p will lie with probability γ . We determine such a γ -confidence interval for $\gamma = 95\%$ confidence level (= safety), or for a 99% confidence level.

In (11) we calculated a γ -estimation interval for h_n for a given P :

$$P\left(|h_n - p| \leq \frac{z \cdot \sigma}{n}\right) \approx 2\Phi(z) - 1 = \gamma.$$

Now we substitute for $\sigma = \sqrt{n \cdot p \cdot (1-p)}$ and receive for any given p the γ -estimate for

$$h_n: |h_n - p| \leq z \cdot \sqrt{\frac{p \cdot (1-p)}{n}}.$$

i.e. with a certainty of γ is $h_n \in \left[p - z \cdot \sqrt{\frac{p \cdot (1-p)}{n}}, p + z \cdot \sqrt{\frac{p \cdot (1-p)}{n}} \right]$. To solve this prob-

lem, we need to find those values for p for which $h_{25} = 0.68$ lies in this interval.

Therefore, the problem is reduced to solving the inequality $|h_n - p| \leq z \cdot \sqrt{\frac{p \cdot (1-p)}{n}}$, with unknown p .

We calculate $z = 1.96$ from $2\Phi(z) - 1 = \gamma = 0.95$. Sample size is $n = 50$.

$$|0.68 - p| \leq 1.96 \cdot \sqrt{\frac{p \cdot (1-p)}{50}}$$

$$(0.68 - p)^2 \leq 1.96^2 \cdot \frac{p \cdot (1-p)}{50}$$

Handwritten calculator solution for the 95% confidence interval problem. The equation entered is $(.68 - p)^2 = \frac{(1.96)^2 \cdot p \cdot (1-p)}{50}$. The calculator returns two solutions: $p = .79241955906$ or $p = .541894428637$.

Printed calculator solution for the 95% confidence interval problem. The equation entered is $\text{solve}\left((0.68 - p)^2 = \frac{(1.96)^2 \cdot p \cdot (1-p)}{50}, p\right)$. The calculator returns two solutions: $p = 0.541894$ or $p = 0.79242$.

Thus, with a confidence level of 95% is p in the interval $[0,541; 0,793]$. With 95% certainty, the percentage of girls in the population may be predicted to be between 54.1% and 79.2%.

If we calculate on the 99% certainty level, i.e. $2\Phi(z) - 1 = 0,99$ and thus $z \approx 2.58$, we can predict with 99% certainty that the percentage of girls in the basic population is between 49.7% and 82.1%.

The greater the statistical certainty, the less precise the statement about the probability, i.e. the wider the confidence interval.

The confidence interval is often also calculated with an approximation method: since one can assume with a sufficiently large sample size that p will be approximately h_n one replaces p by h_n on the right side of $|h_n - p| \leq z \cdot \sqrt{\frac{p \cdot (1-p)}{n}}$.

$$|h_n - p| \leq z \cdot \sqrt{\frac{p \cdot (1-p)}{n}}$$

$$|h_n - p| \leq z \cdot \sqrt{\frac{h_n \cdot (1-h_n)}{n}}$$

$$|0.68 - p| \leq 1.96 \cdot \sqrt{\frac{0.68 \cdot 0.32}{50}} \approx 0.13$$

This means that p would lie in the interval $[0.55; 0.81]$ with 95% confidence. This approximation provides good results for large n and $p \approx 0,5$ only. A rule of thumb for their applicability is $0.3 < p < 0.7$. Therefore, the approximation in our particular example is not so good.

- (15) Let's now calculate the 95 % confidence intervals for different sample sizes from (13).

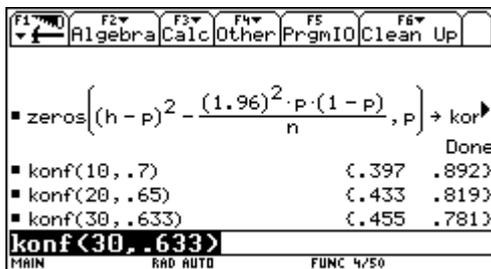


Figure 54

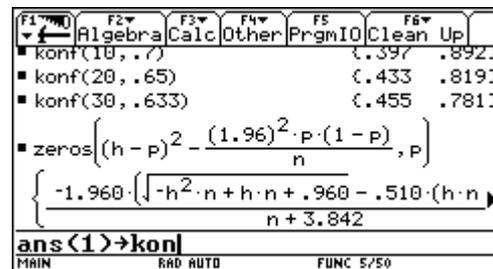
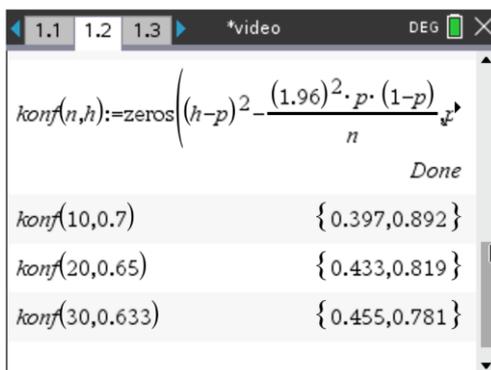
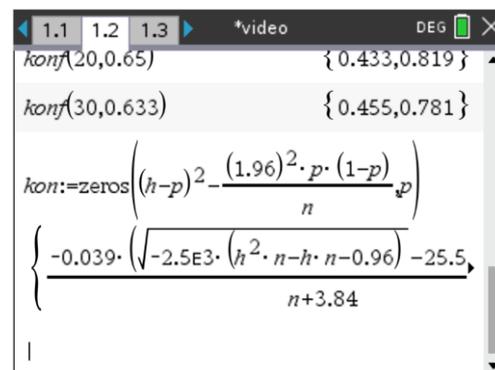


Figure 55



N-Figure 54



N-Figure 55

Use the TI92 to calculate the zeros of the function $(h-p)^2 - \frac{1.96^2 \cdot p \cdot (1-p)}{n}$ (as a function of p) and then save the resulting bounds of the confidence interval in a list as **konf(n,h)**.

In our case, it is even more practical to receive the results in a data sheet. To do this, save the list of results of the general calculation as **kon**. Then **kon[1]** specifies the first number of the list and **kon[2]** the second one.

In the data sheet, the sample sizes should appear in the first column and the relative frequencies in the second column. By entering **c3 = kon[1]|n = c1** and **h = c2** and **c4 = kon[2]|n = c1** and **h = c2**, we obtain the bounds of the confidence intervals in the corresponding columns.

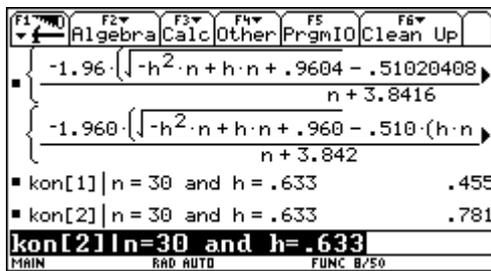


Figure 56

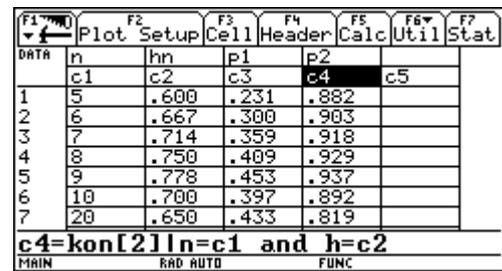
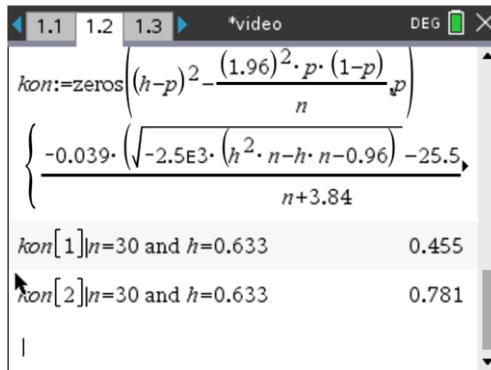


Figure 57



N-Figure 56



N-Figure 57

In the TI-Nspire enter the expressions for the bounds in cells *p1* and *q1*, respectively and then copy the cells down (see N-Figure 57).

Sample size	relative frequency of girls in the sample	95% - confidence interval for the girls' proportion in the population	
5	0.600	0.321	0.882
6	0.667	0.300	0.903
7	0.714	0.359	0.918
8	0.750	0.409	0.929
9	0.778	0.453	0.937
10	0.700	0.397	0.892
20	0.650	0.433	0.819
30	0.633	0.455	0.781
40	0.800	0.652	0.895
50	0.680	0.542	0.792
60	0.667	0.541	0.773
70	0.600	0.483	0.707
80	0.638	0.529	0.735
90	0.667	0.565	0.756
100	0.640	0.542	0.727
110	0.672	0.580	0.753
120	0.658	0.587	0.752
130	0.646	0.561	0.723

If we plot the upper and lower bounds of the 95% confidence intervals as a function of sample size, we see that the confidence intervals become smaller and smaller with increasing sample size, but also depend on the size of the frequency.

In addition, the (usually unknown) proportion p of girls in the population is plotted.

In figure 58, the 99% confidence interval has also been plotted. (We use list **kon99**, which is obtained by replacing 1.96 by 2.58 in list **kon** from above.)

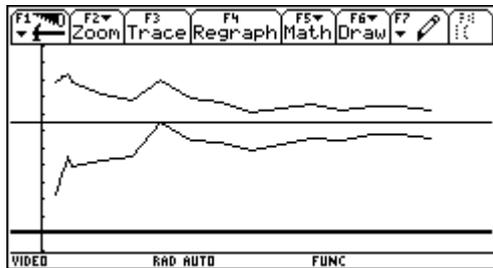


Figure 57

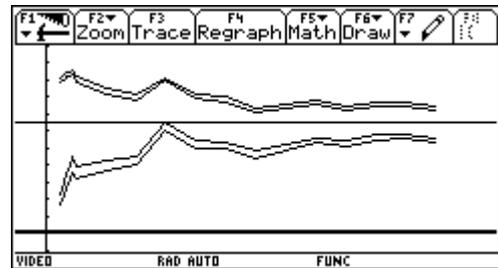
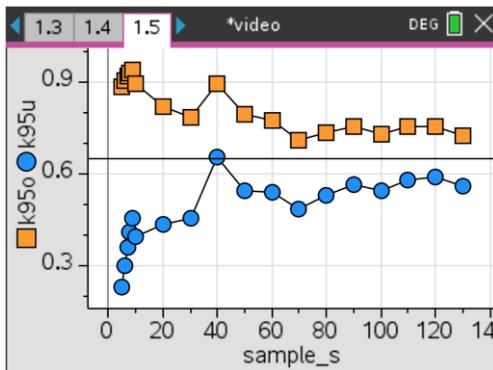
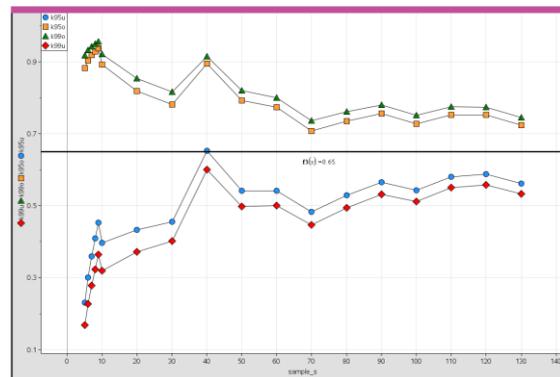


Figure 58



N-Figure 57



N-Figure 58

- (16) Based on a preliminary investigation it is known that there are about 70% girls in the school. A new survey shall re-estimate this percentage, with 95% confidence. The prescribed length of the confidence interval should have the specified length of 0.2, i.e. p is determined with an estimate with a maximum deviation of $\pm 10\%$.

Half-length of the confidence interval is approximately given by $\varepsilon = z \cdot \sqrt{\frac{h \cdot (1-h)}{n}}$. From

this follows that $n = \frac{z^2 h \cdot (1-h)}{\varepsilon^2} = \frac{1.96^2 \cdot 0.7 \cdot 0.3}{0.1^2} \approx 81$. This means that 81 persons must

be interviewed in order to estimate the proportion of girls with 95% probability with a maximum deviation of $\pm 10\%$. If we wanted to increase the accuracy to $\pm 5\%$, then 323 samples would be necessary. In our example, this is already more than the population.

Likewise, the confidence interval changes when h is changed. If no value for h is known at all, one must assume the least favorable value.

How does the sample size depend on h ?

If the formula is stored as a function of the sample size, a parabola with a maximum at 50% is obtained (zeros are 0 and 1!). This means that in the worst case, the sample size

would be at maximum deviation with 95% probability $\frac{1.96^2 \cdot 0.5^2}{0.1^2} \approx 97$.

(17) How many people need to be surveyed in a large-scale survey to determine their preferences for certain types of computer games if the survey is to be accurate with 99% confidence and

- a) no data are known,
- b) the results of (8) are known?

$$\frac{(2.58)^2 \cdot h \cdot (1-h)}{(.01)^2} \quad | \quad h = 0.5 \quad .255 \quad .328$$

16641.	12645.	14672.	8626.	12414
5.	255.	328.	153.	248.
686				

MAIN DEG AUTO FUNC 1/30

No data are known	Action	Adventure	Simulation	Sport	Strategy
50%	25.5%	32.8%	15.3%	24.8%	68.6%
16 641	12 645	14 672	8 626	12 414	14 338

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On the Trace of the Spider

Josef Böhm, Austria

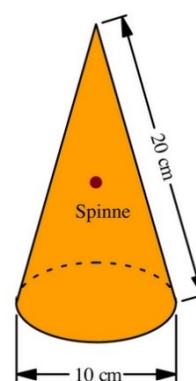
It was in November when I browsed through the nice and inspiring puzzles provided by Heinrich Hemme (<https://www.spektrum.de/raetsel/>). There was one (from 28 July 2021): “How long is the shortest way?” BTW, it was Wolfgang Alvermann who drew my attention to these puzzles some time ago.

This is a problem which can be found originally in a book written by the famous Polish mathematician Hugo Steinhaus (1887–1972) in his book “Kalejdoskop Matematyczny”, which was translated into many languages (e.g. German: “Kaleidoskop der Mathematik”, English: “Mathematical Snapshots”).

This is the problem:

At the half of the slant height of a circular cone with base diameter 10 and slant height 20 sits a spider. It crawls around the cone once and gets back to its starting point. Its distance from the tip of the cone does not necessarily always remain the same. By chance it has taken the shortest possible path. How long is this path?

(<https://www.spektrum.de/raetsel/wie-lang-ist-der-kuerzeste-weg/1896961>).



The solution is very easy and does not require any tool. Do you find the length of the path?

I had the idea to produce a 3D-model of the cone together with the path of the spider applying sliders for changing the dimensions of the cone. (You find a picture of my paper model on page 3.)

So, I start modelling the problem in the plane: I roll up the surface of the cone which turns out to be the quarter of a circle ($r = 20$, $\text{arc} = 10\pi \rightarrow \alpha = 90^\circ$). If you cut the surface along a slant height through the point marking the spider, then you can easily read off the distance as the diagonal of a square (see the following DERIVE construction).

I generalize with slant height s_- and angle a of the sector given (instead of the diameter d). So, we need a sector with radius s_- and angle a . The center of the circle is S .

I install sliders for s_- with $0 \leq s_- \leq 20$ (50 intervals), a with $0 \leq a \leq 2\pi$ (32 intervals) and parameter t with $0 \leq t \leq 1$ (for plotting the arc).

$$S := [0, s_-]$$

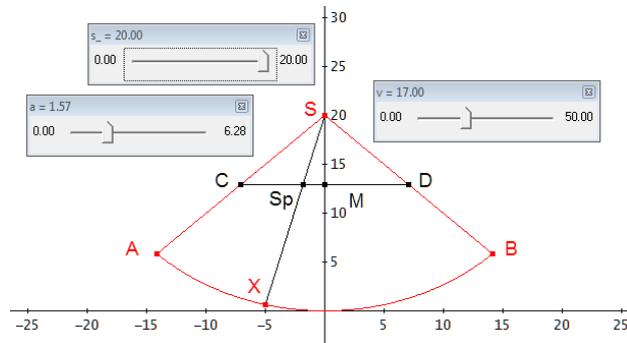
$$A := \left[s_- \cdot \cos\left(\frac{3 \cdot \pi}{2} - \frac{a}{2}\right), s_- + s_- \cdot \sin\left(\frac{3 \cdot \pi}{2} - \frac{a}{2}\right) \right]$$

$$B := \left[s_- \cdot \cos\left(\frac{3 \cdot \pi}{2} + \frac{a}{2}\right), s_- + s_- \cdot \sin\left(\frac{3 \cdot \pi}{2} + \frac{a}{2}\right) \right]$$

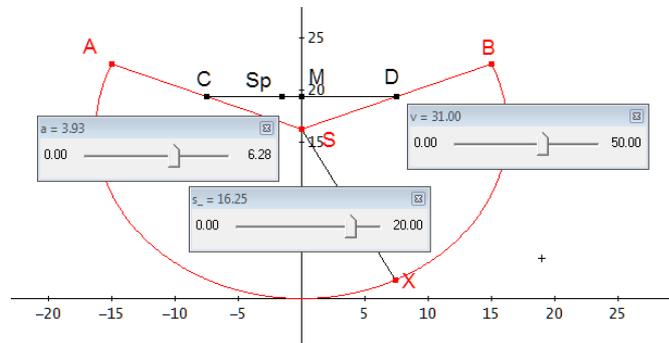
$$\text{base} := \left[s_- \cdot \cos\left(\frac{3 \cdot \pi}{2} - \frac{a}{2} + t \cdot a\right), s_- + s_- \cdot \sin\left(\frac{3 \cdot \pi}{2} - \frac{a}{2} + t \cdot a\right) \right]$$

Then I define points C and D (start and endpoint of the spider's tour around the cone) and point X on the base of the circle. X can be moved in 50 steps by the slider for v ($0 \leq v \leq 50$). Sp is the position of the spider on the rolled-up surface.

This is the position belonging to the problem:



And this is a position with angle $a > \pi$.



Respective calculations can be found in `Spider_plane.dfw`.

Now let's turn to the 3D model and investigate the space curve of the crawling spider. I really must mention, that Wolfgang Alvermann's treatment was a great support because I could refer to his results. Wolfgang introduced the slant height and the diameter of the cone as variables^[*]. I took slant height and angle of the sector forming the surface of the cone. I decided to work in degrees!!

Main problem is to take care that the run of the spider = point Sp (variable $0 \leq u \leq a$) is synchronous with the run of point X . ($u = \angle CSSp$).

#1: `[CaseMode := Sensitive, s := Real [0, ∞)]`

$$\#2: \left[r(s, a) := \frac{a \cdot s}{360^\circ}, h(s, a) := \sqrt{(s^2 - r(a, s)^2)} \right]$$

#3: `T(s, a) := [0, 0, h(s, a)]`

Base of the cone followed by the cone as wire grid

$$\#4: BP(s, a) := \left[r(s, a) \cdot \cos\left(\frac{s \cdot u^\circ}{r(s, a)}\right), r(s, a) \cdot \sin\left(\frac{s \cdot u^\circ}{r(s, a)}\right), 0 \right]$$

$$\#5: \text{VECTOR}\left(BP(s, a), u, 0, a, \frac{a}{100}\right)$$

$$\#6: \text{VECTOR}\left([\text{[T(s, a), BP(s, a)]}], u, 0, a, \frac{a}{18}\right)$$

[*] Wolfgang's treatment (in German) is included in MTH128.zip. Thanks, Wolfgang.

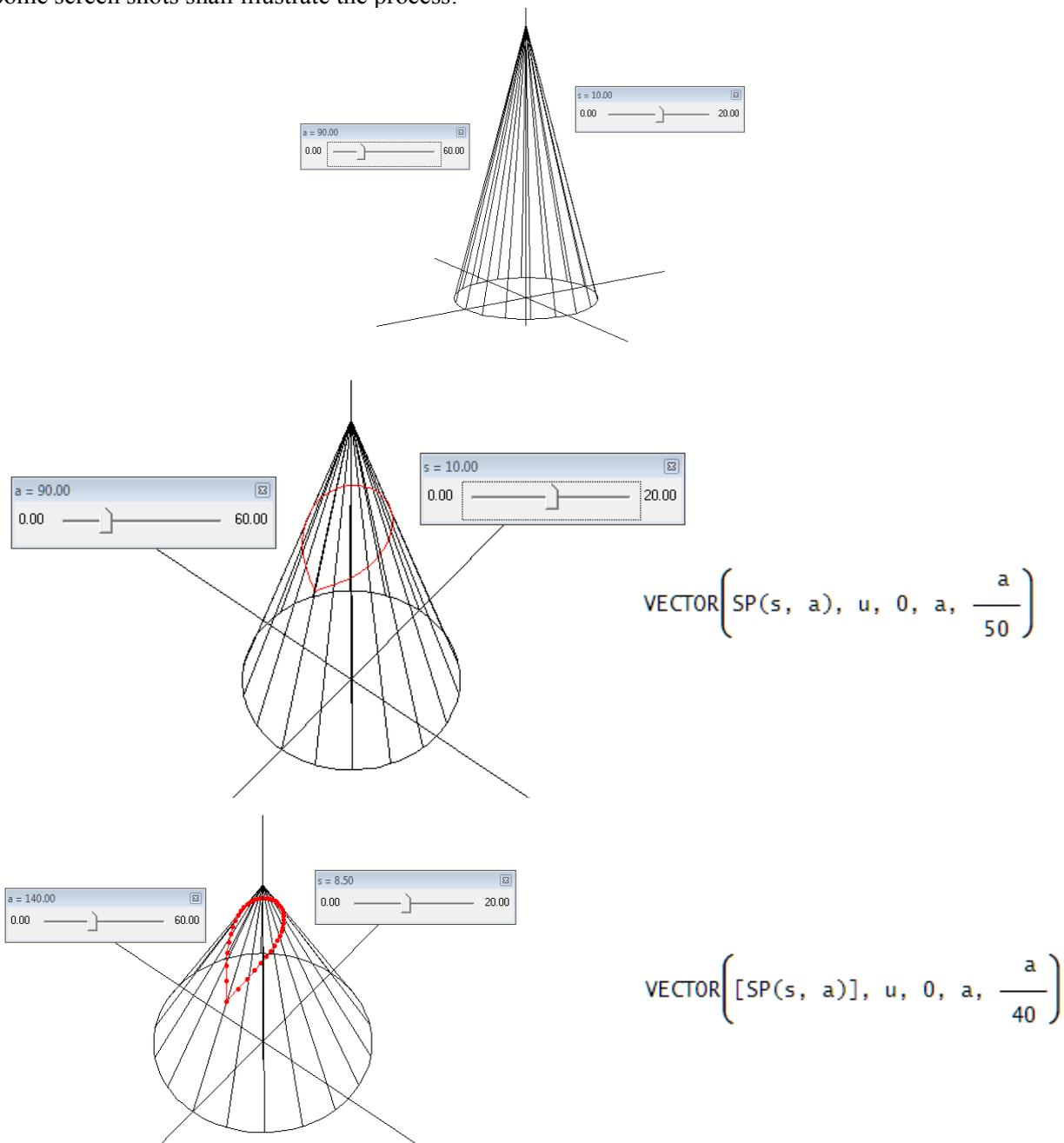
TSp is the distance between the top of the cone and the position of the spider depending on u :

$$TSp(s, a) = \frac{s \cdot \cos\left(\frac{\pi \cdot a}{360}\right)}{2 \cdot \cos\left(\frac{\pi \cdot a}{360} - \frac{\pi \cdot u}{180}\right)}$$

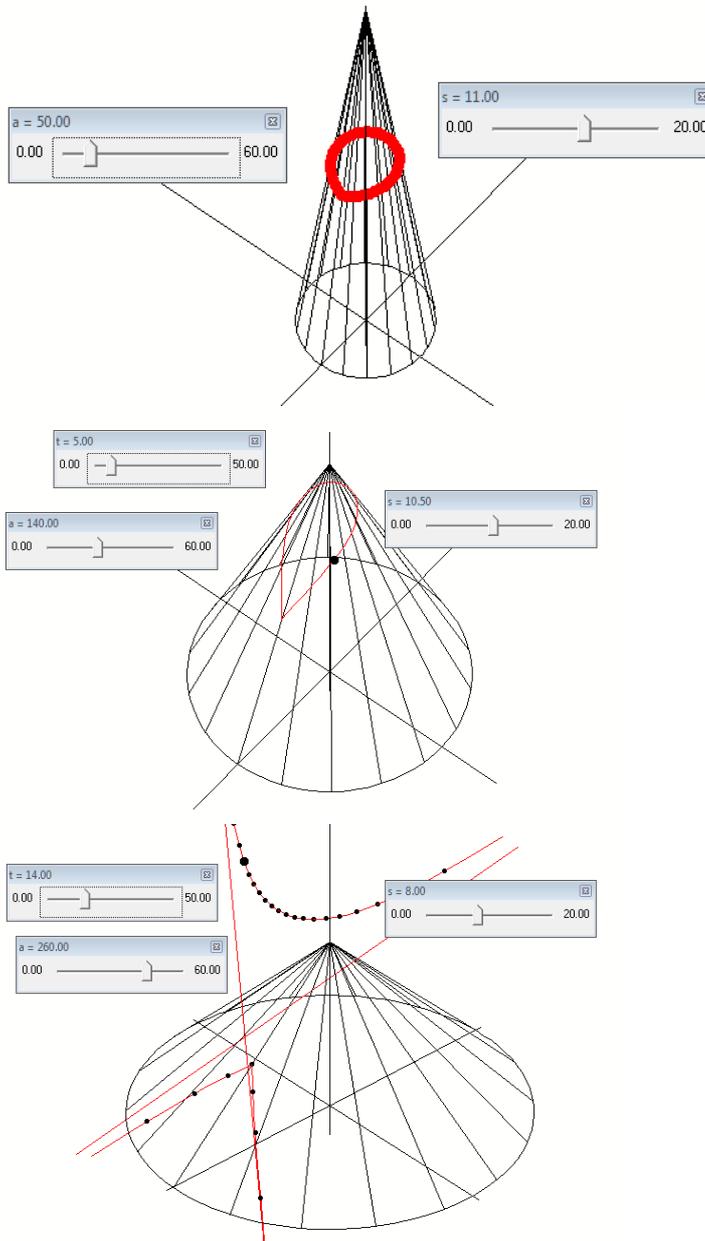
Then, $SP(s, a)$ is the position of the spider on the surface of the cone which describes the space curve:

$$\#10: SP(s, a) := T(s, a) + \frac{TSp(s, a) \cdot (BP(s, a) - T(s, a))}{s}$$

Some screen shots shall illustrate the process:



The trace of the spider as a closed line (above) and as discrete points in medium size (below).



And now in large size discrete points.

I introduce a parameter $0 \leq t \leq 50$ by replacing u by $t/50$ in order to simulate the spider's trace in 50 steps by moving the slider.

How does the space curve look for an angle $a > 180^\circ$?

Strange, isn't it? This is one example.

Wolfgang had a nice idea. He wrote:

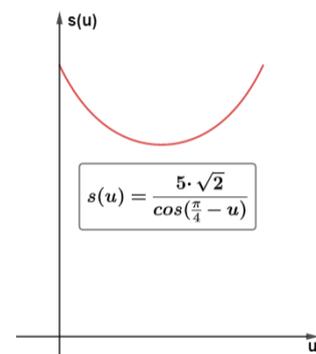
You described the distance from the cone tip to the spider's path as $s(u) = \frac{5 \cdot \sqrt{2}}{\cos\left(\frac{\pi}{4} - u\right)}$. I am

quite sure that you have noticed that there is a small optimization problem hidden (to be honest, I did not, Josef).

$$s'(u) = -5 \cdot \sqrt{2} \cdot \frac{\sin\left(\frac{\pi}{4} - u\right)}{\cos^2\left(\frac{\pi}{4} - u\right)} \rightarrow u = \frac{\pi}{4} \quad \dots \text{as expected!!}$$

$$s'(u) = 0$$

So, $s = 5 \cdot \sqrt{2}$ zum Punkt $P = \left(-\frac{5}{4}\sqrt{2}/0 / \frac{5}{4}\sqrt{15} \cdot (4 - \sqrt{2})\right)$



Finally, let's have a little bit calculus: We will check all our work by calculating the length of the space curve.

It is necessary to convert u :

$$\text{check} := \text{SUBST}\left(\text{SP}(20, 90), u, \frac{u}{1^\circ}\right)$$

$$\text{check} := \left[\frac{5 \cdot \sqrt{2} \cdot \cos(4 \cdot u)}{4 \cdot \sin\left(u + \frac{\pi}{4}\right)}, \frac{5 \cdot \sqrt{2} \cdot \sin(4 \cdot u)}{4 \cdot \sin\left(u + \frac{\pi}{4}\right)}, 5 \cdot \sqrt{15} - \frac{5 \cdot \sqrt{30}}{4 \cdot \sin\left(u + \frac{\pi}{4}\right)} \right]$$

$$\int_0^{\pi/2} \sqrt{\left(\frac{d}{du} \text{check}_1\right)^2 + \left(\frac{d}{du} \text{check}_2\right)^2 + \left(\frac{d}{du} \text{check}_3\right)^2} du = 10 \cdot \sqrt{2}$$

Other angles:

$$\text{check60} := \text{SUBST}\left(\text{SP}(20, 60), u, \frac{u}{1^\circ}\right)$$

$$\text{check60} := \left[\frac{5 \cdot \sqrt{3} \cdot \cos(6 \cdot u)}{6 \cdot \sin\left(u + \frac{\pi}{3}\right)}, \frac{5 \cdot \sqrt{3} \cdot \sin(6 \cdot u)}{6 \cdot \sin\left(u + \frac{\pi}{3}\right)}, \frac{10 \cdot \sqrt{35}}{3} - \frac{5 \cdot \sqrt{105}}{6 \cdot \sin\left(u + \frac{\pi}{3}\right)} \right]$$

$$\int_0^{\pi/3} \sqrt{\left(\frac{d}{du} \text{check60}_1\right)^2 + \left(\frac{d}{du} \text{check60}_2\right)^2 + \left(\frac{d}{du} \text{check60}_3\right)^2} du = 10$$

$$\text{check120} := \text{SUBST}\left(\text{SP}(20, 120), u, \frac{u}{1^\circ}\right)$$

$$\text{check120} := \left[\frac{5 \cdot \cos(3 \cdot u)}{3 \cdot \sin\left(u + \frac{\pi}{6}\right)}, \frac{5 \cdot \sin(3 \cdot u)}{3 \cdot \sin\left(u + \frac{\pi}{6}\right)}, \frac{40 \cdot \sqrt{2}}{3} - \frac{10 \cdot \sqrt{2}}{3 \cdot \sin\left(u + \frac{\pi}{6}\right)} \right]$$

$$\int_0^{2 \cdot \pi/3} \sqrt{\left(\frac{d}{du} \text{check120}_1\right)^2 + \left(\frac{d}{du} \text{check120}_2\right)^2 + \left(\frac{d}{du} \text{check120}_3\right)^2} du = 10 \cdot \sqrt{3}$$

DERIVE file: Spider_3D_deg.dfw

References:

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<https://archive.org/details/kalejdoskop-matematyczny-steinhaus-hugo/page/180/mode/2up>
(in Polish!!)