Intersecting Tangents of a Parabola

Teacher Notes

Introduction

This activity allows students to investigate the properties of the point of intersection of two tangents to a parabolic curve.

Resources

Students start with a new, blank TI-Nspire document. A **worksheet** lays out the task and guides students, step by step, through the necessary construction.

Skills required

It is assumed that students will be able to carry out the following basic TI-Nspire processes.

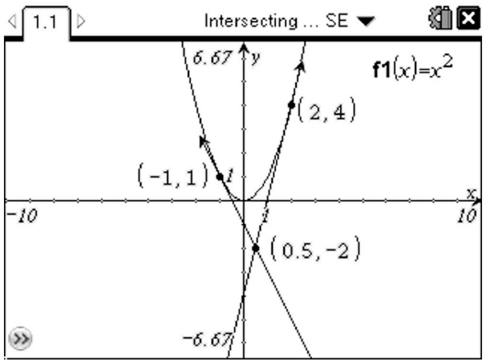
- ✓ Open and save a new tns document.
- ✓ Draw the graph of a function on a Graphs page.
- ✓ Use menus to select commands.
- ✓ Grab and move objects on a Graphs page.
- ✓ Use (ctrl)(menu) to access contextual menus.

Other techniques are described in full on the worksheet.

The activity

Construction

Students are guided, step by step, to create a Graphs page similar to the one shown below.



Gathering data and making predictions

By editing the x-coordinates of the two points on the parabola students explore their numerical effect on the coordinates of the intersection point. They are encouraged to keep a written note of everything that they do and of conclusions that they reach.

The aim is to be able to predict the coordinates of the intersection point when the coordinates of the points on the parabola are (a, a^2) and (b, b^2).

Algebraic analysis

The worksheet leads students through steps necessary to change their prediction into a proof.

i.) They are told that, for f(x)=x^2, the gradient of the line at x=a is 2a. (NB They are not expected to know this fact before tackling the task) They can then find the equation of the line that goes through (a, a^2) with gradient 2a.

ii.) Find the equation of the line that goes through (b, b^2) with gradient 2b.

iii.) Solve these simultaneous equations to find the intersection point, $\left(\frac{a+b}{2}, ab\right)$.

Students are encouraged to check this general result against their numerical data.

Next Steps

Students are encouraged to investigate one of the following extension questions, or one that they pose for themselves.

- What happens when instead of f(x)=x^2, it is f(x)=2x^2 or f(x)=3x^2 ?
- If I knew the coordinates of the intersection point, could I work out possible tangent points on the parabola? *[this is, in effect, the ‘backwards’ version of the process already explored]*
- What happens when instead of f(x)=x^2, it is f(x)=x^3 or f(x)=x^4 ? *[this question leads to much more complex algebraic work.]*