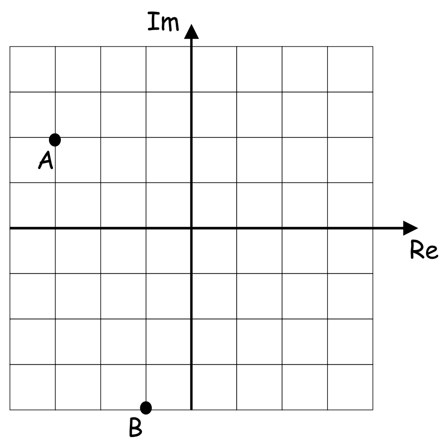
OCR Further Pure 1

Complex Numbers

Section 2: Equations and geometrical representation

Multiple Choice Test

Questions 1 and 2 refer to the Argand diagram below.



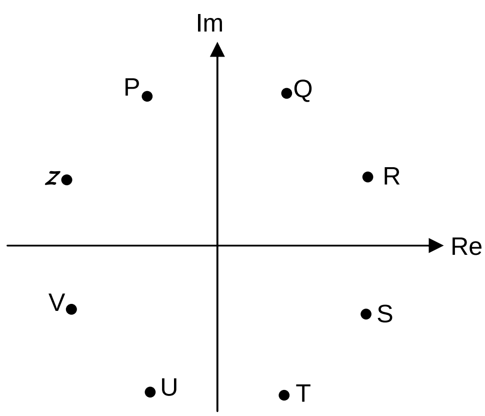
1) In the Argand diagram, the point A represents the complex number

(a) $-3 + 2i$
(b) $3 - 2i$
(c) $2 - 3i$
(d) $-2 + 3i$
(e) I don't know

2) In the Argand diagram, the point B represents the complex number

(a) $-4 - i$
(b) $-1 - 4i$
(c) $1 + 4i$
(d) $4 + i$
(e) I don't know

Questions 3 – 4 refer to the Argand diagram below. The point representing the complex number z is shown on the diagram.



3) The point which represents z^* is

(a) T
(b) R
(c) Q
(d) V
(e) I don't know

4) The point which represents iz is

(a) P
(b) S
(c) U
(d) Q
(e) I don't know

5) 2 + i is a root of $z^3 - z^2 - 7z + 15 = 0$. The other roots are

(a) 2 + i, 3
(b) 2 – i, 3
(c) 2 – i, –3
(d) 2 + i, 2 – i
(e) I don't know

6) The real root of $z^3 - 4z^2 + 14z - 20 = 0$ is 2. The other roots are

(a) –1 + 3i, –1 – 3i
(b) 1 + 3i, 1 – 3i
(c) 2 + 3i, 2 – 3i
(d) –2 + 3i, –2 – 3i
(e) I don't know

7) 1 + 2i is a root of the cubic equation $z^3 + az^2 + bz + 5 = 0$.
The values of a and b are

(a) $a = -1$, $b = 3$
(b) $a = 1$, $b = -1$
(c) $a = 1$, $b = 3$
(d) $a = -1$, $b = -1$
(e) I don't know

8) –2 + i is a root of the equation $z^4 + 2z^3 - z^2 - 2z + 10 = 0$.
The other roots are

(a) –2 – i, 1 + 2i, 1 – 2i
(b) 2 – i, 1 + i, 1 – i
(c) –2 – i, 2 – i, 2 + i
(d) –2 – i, 1 + i, 1 – i
(e) I don't know

9) The equation $z^4 + z^3 + 2z^2 + 4z - 8 = 0$ has two real roots. The roots of the equation are

(a) –1, 2, 1 + i, 1 – i
(b) 1, –2, 2i, –2i
(c) 1, –2, 1 + i, 1 – i
(d) –1, 2, 2i, –2i
(e) I don't know

10) The square roots of the complex number 5 + 12i are

(a) 2 + 3i and –2 – 3i
(b) 3 – 2i and –3 + 2i
(c) 3 + 2i and –3 – 2i
(d) 2 – 3i and –2 + 3i
(e) I don't know

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Solutions to Multiple Choice Test

1) The correct answer is a)

A is the point (-3, 2).
This represents the complex number $-3 + 2i$.

2) The correct answer is b)

B is the point (-1, -4).
This represents the complex number $-1 - 4i$.

3) The correct answer is d)

If $z = x + iy$, then $z^* = x - iy$.
So the point which represents z^* is the reflection of the point which represents z in the x-axis.
This is point V.

4) The correct answer is c)

If $z = x + iy$, then $iz = ix - y = -y + ix$.
The point which represents iz has x-coordinate equal to the y-coordinate of z, but with opposite sign, and y-coordinate equal to the x-coordinate of z.
This is point U.

5) The correct answer is c)

Since $2 + i$ is a root, $2 - i$ is also a root.
So $(z - 2 - i)(z - 2 + i)$ is a factor of the equation.

$$(z-2-i)(z-2+i) = (z-2)^2 + 1$$
$$= z^2 - 4z + 5$$

So $z^3 - z^2 - 7z + 15 = (z^2 - 4z + 5)(z + 3)$
So the third root is -3.
The other two roots are $2 - i$ and -3.

6) The correct answer is b)

2 is a root of the equation, so $z - 2$ is a factor.

$z^3 - 4z^2 + 14z - 20 = 0$

$(z-2)(z^2 - 2z + 10) = 0$

The other two roots are the roots of the quadratic equation $z^2 - 2z + 10 = 0$

$$\begin{aligned} z &= \frac{2 \pm \sqrt{4 - 4 \times 1 \times 10}}{2} \\ &= \frac{2 \pm \sqrt{-36}}{2} \\ &= \frac{2 \pm 6i}{2} \\ &= 1 \pm 3i \end{aligned}$$

7) The correct answer is a)

$(1+2i)^2 = 1 + 4i - 4 = -3 + 4i$

$(1+2i)^3 = (-3+4i)(1+2i) = -3 - 2i - 8 = -11 - 2i$

Substituting into $z^3 + az^2 + bz + 5 = 0$:

$-11 - 2i + a(-3+4i) + b(1+2i) + 5 = 0$

Equating real parts: $-11 - 3a + b + 5 = 0 \Rightarrow 3a - b = -6$

Equating imaginary parts: $-2 + 4a + 2b = 0 \Rightarrow 2a + b = 1$

Adding: $5a = -5 \Rightarrow a = -1, \quad b = 3$

8) The correct answer is d)

$-2 + i$ is a root, so $-2 - i$ is a root

so $(z + 2 - i)(z + 2 + i)$ is a factor.

$$\begin{aligned} (z+2-i)(z+2+i) &= (z+2)^2 + 1 \\ &= z^2 + 4z + 5 \end{aligned}$$

$z^4 + 2z^3 - z^2 - 2z + 10 = (z^2 + 4z + 5)(z^2 - 2z + 2)$

The other two roots are the roots of the quadratic equation $z^2 - 2z + 2 = 0$.

$$\begin{aligned} z &= \frac{2 \pm \sqrt{4 - 4 \times 1 \times 1}}{2} \\ &= \frac{2 \pm \sqrt{-4}}{2} \\ &= \frac{2 \pm 2i}{2} \\ &= 1 \pm i \end{aligned}$$

The other roots are $-2 - i$, $1 + i$ and $1 - i$.

9) The correct answer is b)

$$f(z) = z^4 + z^3 + 2z^2 + 4z - 8$$
$$f(1) = 1 + 1 + 2 + 4 - 8 = 0$$
$$f(-2) = 16 - 8 + 8 - 8 - 8 = 0$$

so $(z - 1)$ and $(z + 2)$ are factors.

$(z - 1)(z + 2) = z^2 + z - 2$

$$z^4 + z^3 + 2z^2 + 4z - 8 = (z^2 + z - 2)(z^2 + 4)$$

The roots of $z^2 + 4 = 0$ are $2i$ and $-2i$.

So the roots of the equation are 1, -2, $2i$ and $-2i$.

10) The correct answer is c)

$$(a + bi)^2 = 5 + 12i$$
$$a^2 + 2abi - b^2 = 5 + 12i$$

Equating real parts: $a^2 - b^2 = 5$

Equating imaginary parts: $2ab = 12 \Rightarrow a = \frac{6}{b}$

Substituting: $\frac{36}{b^2} - b^2 = 5$

$$36 - b^4 = 5b^2$$
$$b^4 + 5b^2 - 36 = 0$$
$$(b^2 + 9)(b^2 - 4) = 0$$
$$b = \pm 2$$

When $b = 2$, $a = 3$

When $b = -2$, $a = -3$

The square roots of $5 + 12i$ are $3 + 2i$ and $-3 - 2i$.