Connecting Algebra, Geometry and Graphs

Lesson Account

In 2008-9 research was carried out in Scottish schools to determine whether the use of TI-Nspire technology to create dynamically linked multiple representations of mathematics would enhance students' understanding, with resulting changes to teachers' classroom practice.

This account by Allan Duncan of the School of Education, University of Aberdeen describes one of 66 lessons that were evaluated as part of the research project.

For more details of the research project see "Teachers' views on dynamically linked multiple representation and relational understanding of mathematics – an investigation into the use of TI-Nspire™ in Scottish secondary schools" published by the University of Aberdeen, Scotland UK, 2010. ISBN 0 902604 77 5.

Teacher

Rona Lindsay

Class

Top set of a half-year group of S2 students working at Levels E/F

Learning Intentions

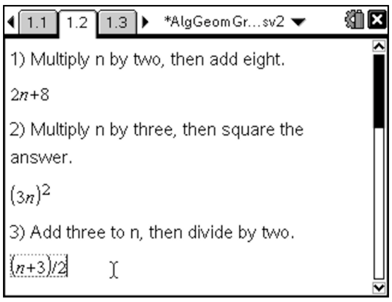
The students would be able to

1. Convert from words to algebra
2. Convert from geometry to algebra
3. Graph algebraic expressions
4. Note the connections between geometry, algebra, graphs and equations

Previous Knowledge

1. Graphs and equations of straight lines and quadratics
2. Algebraic expressions
3. Area of rectangle and square

In this lesson Rona wanted to use a variety of applications within TI-Nspire to try to make connections across different aspects of maths, in particular across algebra, geometry and graphs. She distributed to her students a pre-constructed tns file, starting with a Notes page containing verbal instructions that had to be converted into algebraic expressions.
This screen shows part of page 1.2 already completed by a student.



The full page contained the following instructions, each followed by a dotted box into which students could type their answers.

1) Multiply n by two, then add eight.
2) Multiply n by three, then square the answer.
3) Add three to n, then divide by two.
4) Divide n by two, then add six.
5) Add two to n, then square the answer.
6) Add four to n, then multiply by two.
7) Square n, then add four.
8) Square n, then multiply by nine.
9) Square n, then add n multiplied by four, then add four.
10) Square n, then multiply by three.

Rona explained carefully to the class what they were expected to do and also issued a worksheet of instructions as follows;

“Problem 1, Page 1.1

In the dotted boxes input an algebraic expression that matches the words – be careful to do this accurately, using brackets if necessary.

Problems 2 to 6

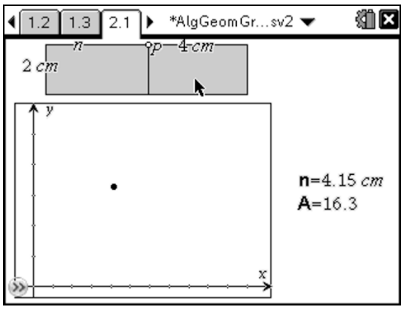
These problems all have a rectangle (split into smaller areas) with a point P which can be grabbed and moved in one direction.

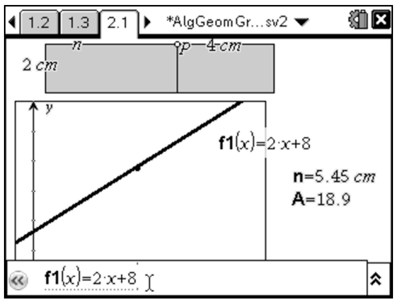
Each page also has a graph with a point plotted – the point represents the *area* plotted against *n,* so as *n* changes the *area* of the whole rectangle changes.

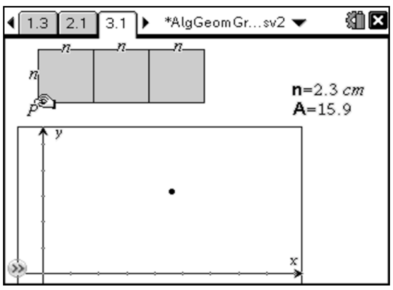
Move point P and observe how the area changes. In addition, observe how the point on the graph moves. Note down your observations on the worksheet. Now try to match one or more of the original expressions on Page 1.1 to the area of each rectangle.

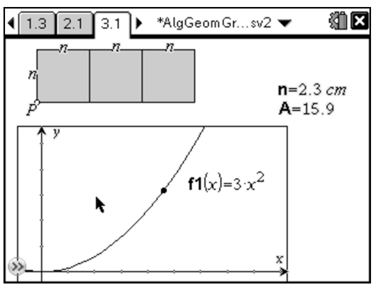
Lastly, input the expression that you think represents the area in the entry line (press ctrl G to get the entry line visible). Then move point P and, if you are correct with your matching, the point on the graph will move along the line or curve you have input.”

Here are two problems completed correctly.





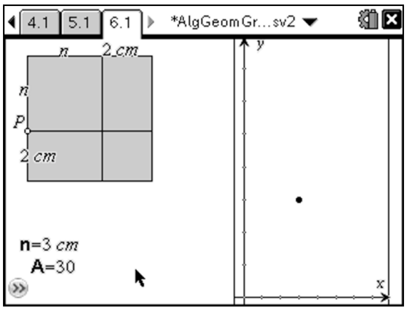


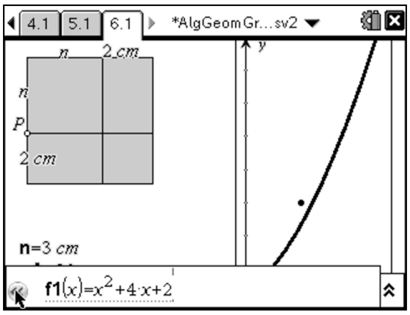


In each of these cases, dragging point P makes the dot move along the curve. The first is the linear equation A = 2*n* + 8, the second is the quadratic A = $3n^2$.

When entering the equation of a function for a graph, TI-Nspire requires the use of x rather than n but the students appeared to accept this and to be quite comfortable translating to and fro between, for example, A = 2n + 8 and f1(x) = 2x + 8.

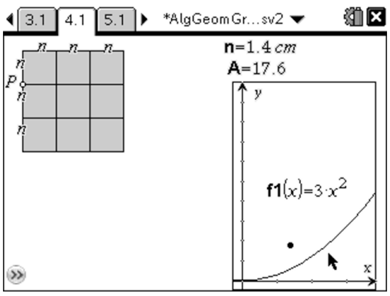
In the next example, the conjectured equation is incorrect and the dot does not appear on the curve.

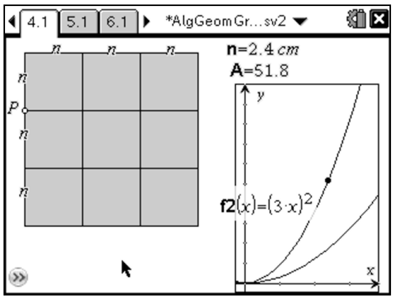




In this example, the student entered the function as f1(x) = x^2 + 4x + 2 and dragging P made the point move very close to but not along the curve. Noticing that the point was always just above the graph, she returned to her geometrical diagram and noticed that the small square is actually 2 by 2 and hence has area 4 square units. She then entered f1(x) = x^2 + 4x + 4 and was able to correct her mistake.

Another possible error would be to write A = $n + 2^2$ rather than A = $(n + 2)^2$. Rona pointed out that the written representations appear very different but this common error results from the way we 'speak' these two expressions. We need to make a distinction between them verbally. The same goes for $3n^2$ and $(3n)^2$ which also arose in this investigation. ("Three n squared" and "three n all squared".)





In this case the correct graph could be entered as y = $(3x)^2$ or as y = $9x^2$. The incorrect graph is y = $3x^2$.

Teacher's comment

By the end of the lesson, most pupils could make the connection between the expressions and the areas of the rectangles (with common misconceptions tackled i.e. $3n^2$ does not equal $(3n)^2$.) Most had discovered that the path of the plotted point was either a straight line or a curve and could give a reason as to why they would know, from the expression, which it would be. And they could also explain the difference between the two types of rectangle and link with the type of expression and line/curve, i.e. rectangle with n in one dimension produced a line, rectangle with n in both dimensions produced a curve.

